3D Vision

BLG634E - Spring 2022

28.02.2022

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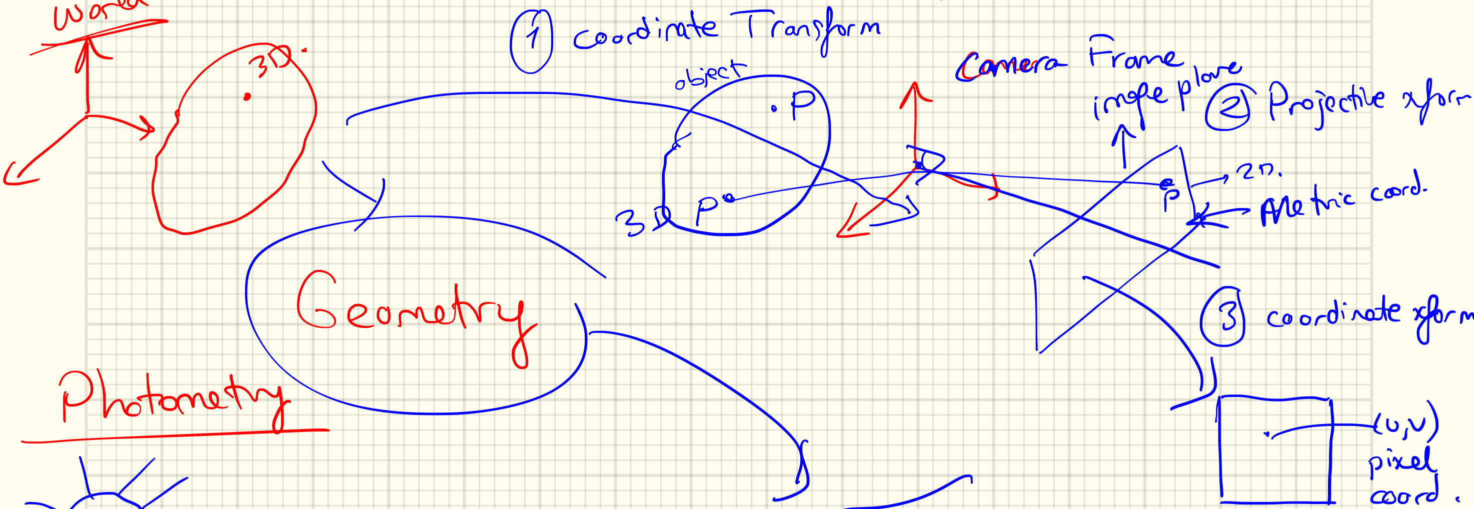
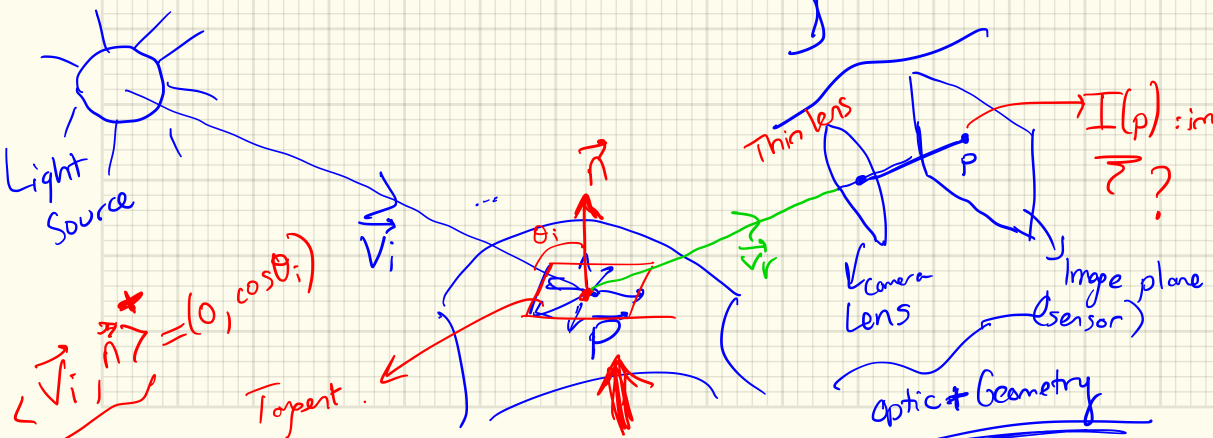
Image Formation Model : → 1) Geometry ± optics.) 3D → 2D.
2) Photometry



Geometry

Photometry



$I(p)$: image irradiance (intensity)

? ?

$\langle \vec{V}_i, \vec{n} \rangle = (0, \cos\theta_i)$

optic + Geometry

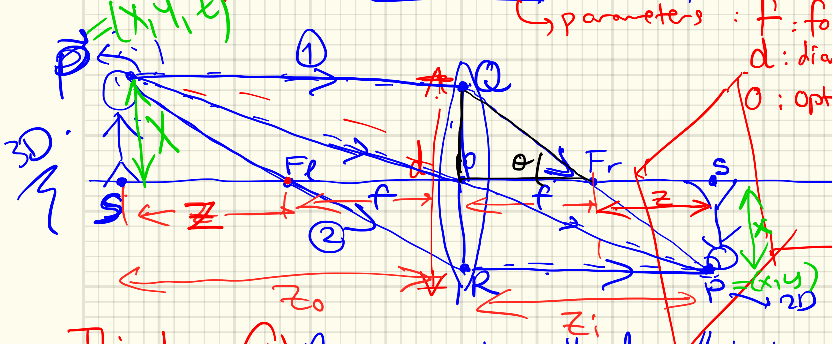
Basic Optics (Thin Lens Model)

$3D \rightarrow 2D$.

parameters: f: focal length
d: diameter of the lens
O: optical center

$$F_{\#} = f/d$$

$$FOV = \theta = \mathrm{atan}\left(\frac{d}{2f}\right)$$



Thin Lens: (1) Any ray entering the lens // to the optical axis on one side goes thru the focus on the other side.

(2) Any ray entering the lens from the focus on one side emerges // to the axis on the other side.

To derive the thin lens equation, use similar triangles

(1) $\langle P, F_l, S \rangle \bowtie \langle R, F_l, O \rangle$

(2) $\langle p, s, F_r \rangle \bowtie \langle Q, O, F_r \rangle$

(1) $\dfrac{Z}{X} = \dfrac{f}{x} \rightarrow \dfrac{X}{x} = \dfrac{Z}{f}$

(2) $\dfrac{z}{x} = \dfrac{f}{X} \rightarrow$ (1)×(2) $Zz = f^2$

Set $z_o = Z + f$, $z_i = z + f$ $\Big\}$ $f^2 = (z_o - f)(z_i - f)$

$\cancel{f^2} = z_o z_i - f(z_o + z_i) + \cancel{f^2}$

$\rightarrow$

$\Rightarrow Z_o z_i = f(Z_o + z_i) \rightarrow \dfrac{1}{f} = \dfrac{Z_o + z_i}{Z_o z_i}$

$\rightarrow$ Fundamental Eqn of Thin Lens : $\boxed{\dfrac{1}{f} = \dfrac{1}{Z_o} + \dfrac{1}{z_i}}$ $\rightarrow$ focusing

Pinhole Camera : (Recall Occam's razor)

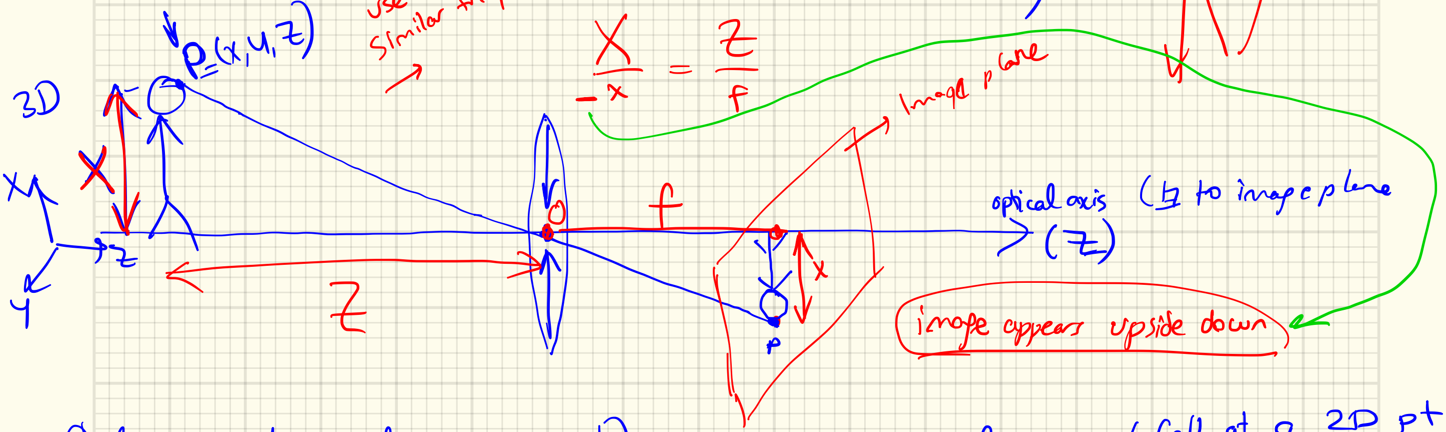
Reduce camera's aperture (lens) to a point (pinhole)

shrink thin lens to 0.

$d \searrow 0$

use similar triangles

$$\frac{X}{-x} = \frac{Z}{f}$$

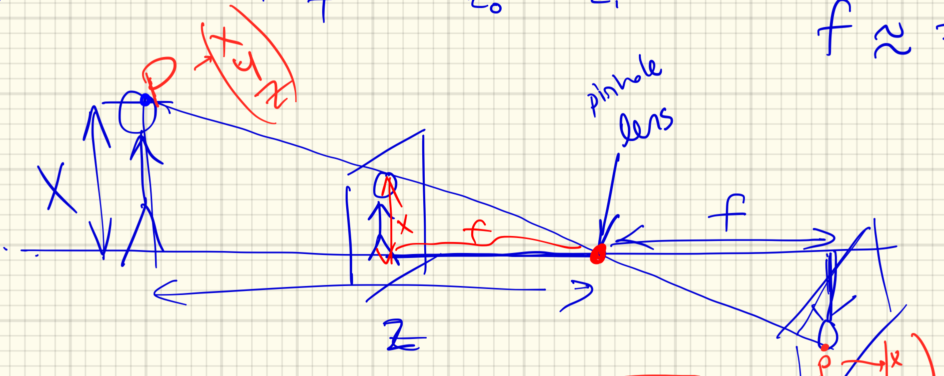


Only a single ray from a point P can enter the camera, focus at p (fall at a 2D pt p on the sensor plane)

Thin lens eqn $\qquad \frac{1}{f} = \frac{1}{Z_0} + \frac{1}{Z_i}$

Pinhole $\quad Z_0 \nearrow \qquad \frac{1}{f} = \frac{1}{Z_i}$

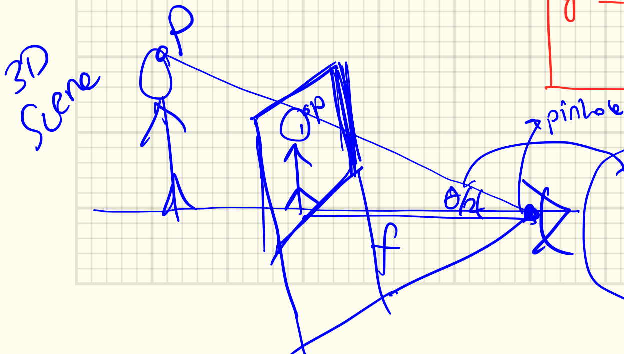
$$f \approx Z_i$$



$$p = \begin{pmatrix} x \\ y \\ f \end{pmatrix}$$

$$\frac{X}{Z} = \frac{x}{f} \longrightarrow \boxed{\begin{aligned} x &= f\,\frac{X}{Z} \\ y &= f\,\frac{Y}{Z} \end{aligned}} \longleftarrow P = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

perspective eqn

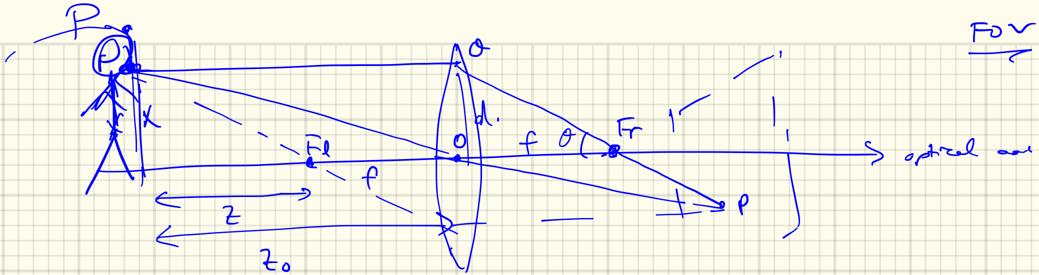


$$\frac{FOV}{2} \; (\theta/2) = \operatorname{atan} \frac{w}{2f}$$

$w \leftarrow$ width of the sensor plane in metric coord.

$FOV = \operatorname{atan}\left(\frac{d}{2f}\right)$

$f \uparrow \quad FOV \downarrow$

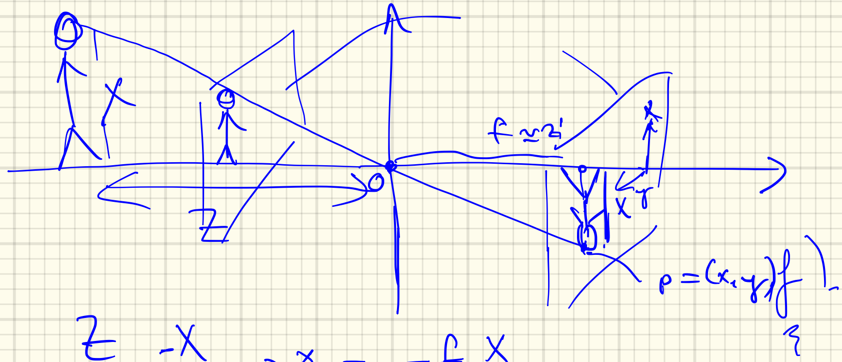


$d \to 0.$

$Z_0 \uparrow$

$$\frac{1}{f} = \frac{1}{Z_0} + \frac{1}{Z_i}$$

$f \approx Z_i$



$$\frac{Z}{f} = \frac{-X}{x} \to x = -f\,\frac{X}{Z}$$

$$y = -f\,\frac{Y}{Z}$$

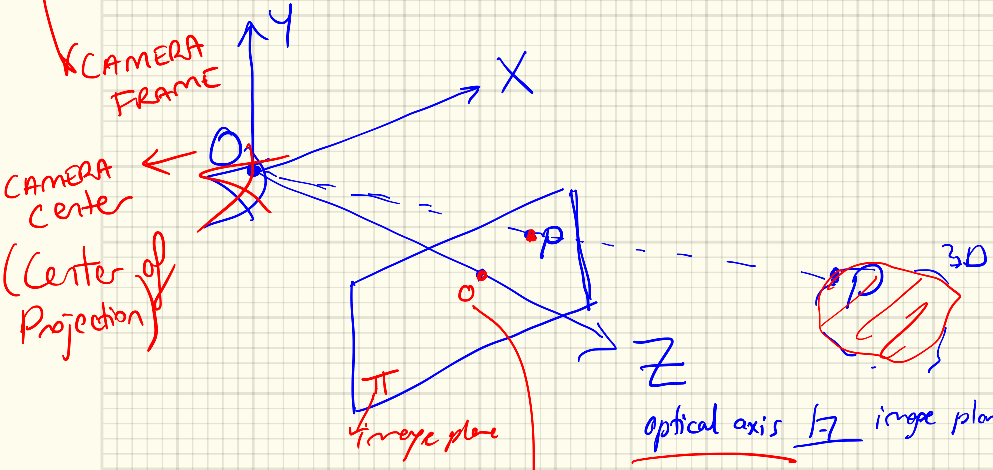
→ Frontal pinhole (Perspective) camera model:

Ideal perspective projection $\underline{P} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \longrightarrow \underline{p} = \begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{Z}\begin{bmatrix} X \\ Y \end{bmatrix}$

$\pi : \mathbb{R}^3 \to \mathbb{R}^2$

projection as a map

→ Nonlinear mapping due to division by depth (factor $\frac{1}{Z}$)

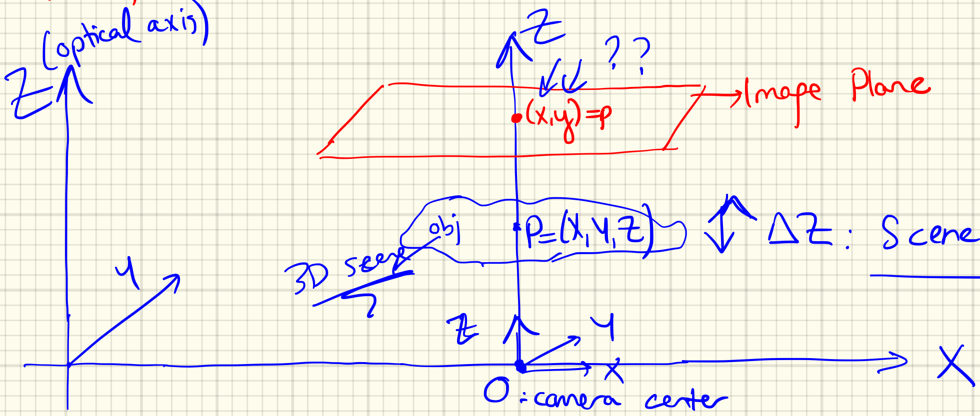


optical axis ⊥ image plane

principal point (Image center) = Intersection of π & the optical axis

Weak Perspectivity

① Weak Perspective Camera Model : Linearize this nonlinear mapping



ΔZ : Scene Depth (relative distance along optical axis of any 2 scene points)

Assumption: Scene's depth is much smaller than the average distance $\bar{Z}$ of the points from the camera center.

$(\Delta Z \ll \bar{Z})$

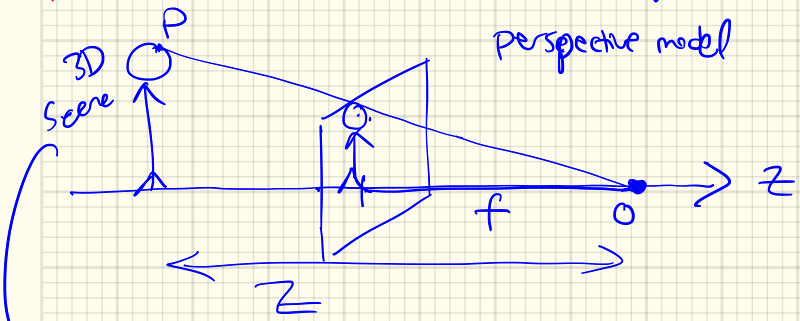
(e.g. a rule of thumb $\Delta Z < \frac{\bar{Z}}{20}$)

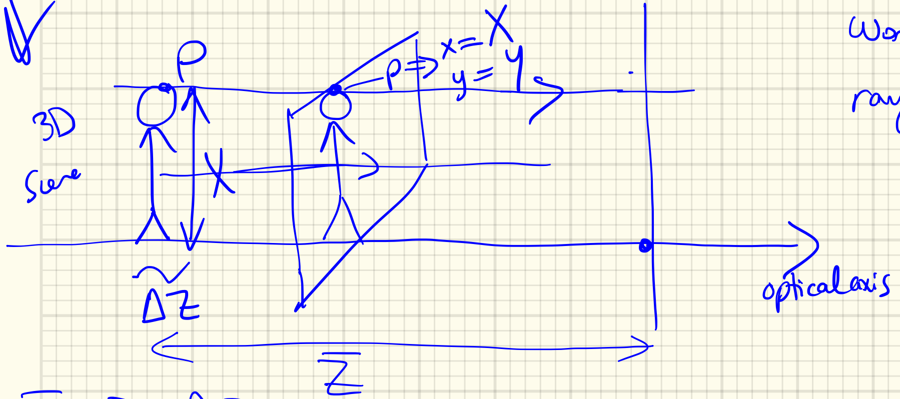
$$x = f\,\frac{X}{Z} \approx f\,\frac{X}{\bar{Z}} \quad \bar{Z} \to \text{const. } \gamma$$

$$y = f\,\frac{Y}{Z} \approx f\,\frac{Y}{\bar{Z}} \quad \bar{Z} \to \gamma$$

2) Orthographic Projection : Limiting model :

$$f \to \infty \qquad Z \to \infty \qquad \frac{f}{Z} \approx 1$$





Orthographic projection

World points are projected along rays parallel to the optical axis

$$x = X$$
$$y = Y$$

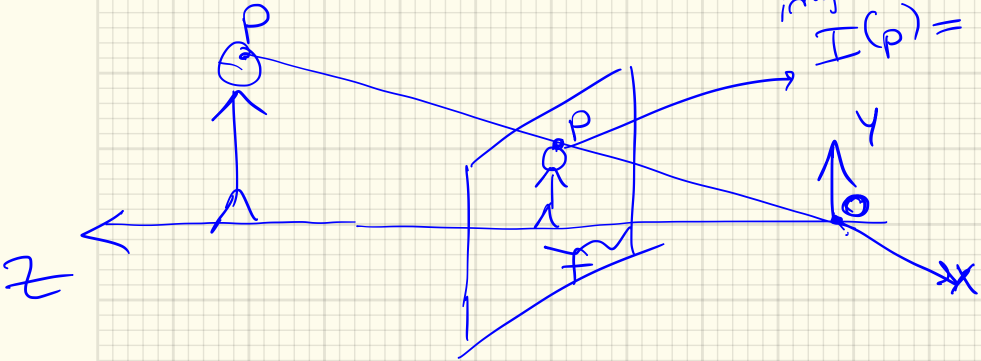
$$Z \gg \Delta Z$$

f & Z are both much further away from the camera center.

image irradiance at $\underline{p}$

$$I(p) = \; ? \quad \propto \quad L(P)$$

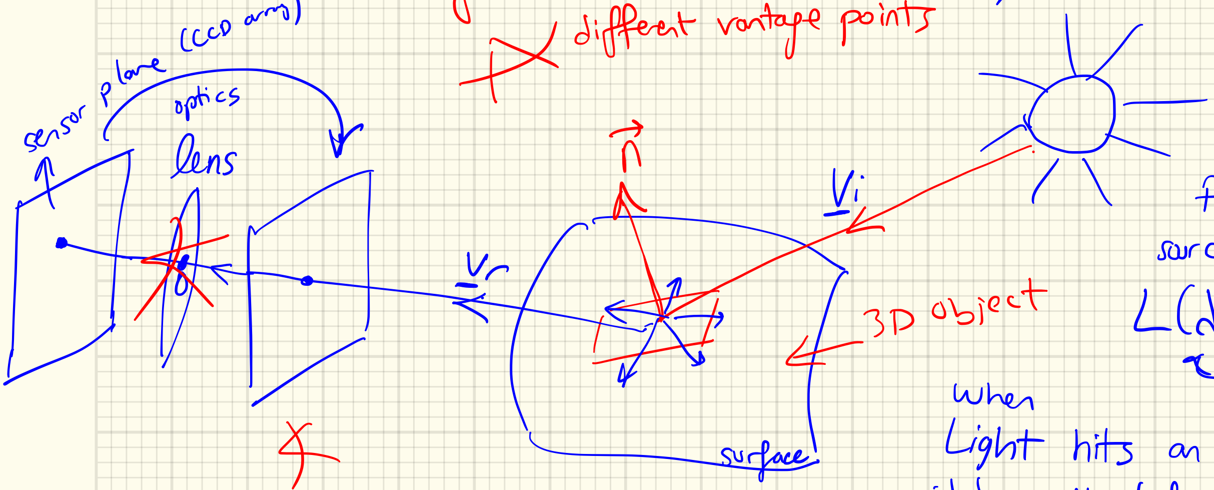
scene radiance at 3D pt $\underline{P}$.



$\downarrow$

Photometric

Photometric Image Formation: (Simplified)



A point light source w/ intensity $L(\lambda)$, i.e. a color spectrum

When Light hits an object surface, it's scattered & reflected.

BRDF: Bidirectional Reflectance Distribution Function.

describe how much of each wavelength arriving at an incident direction $\underline{v_i}$ is emitted in a reflected direction $\underline{v_r}$.

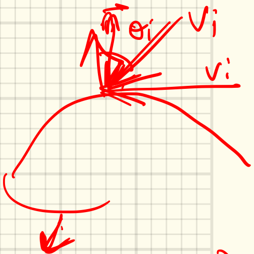
BRDF: $f(\underline{v_i}, \underline{v_r}, \underline{n}, \lambda)$ — λ: wavelength of light

Light source w/ intensity $L(\underline{v_i}, \lambda)$: incident light →

→ To calculate amount of light exiting a surface at point P in a direction $\underline{v}_r$, we integrate the product of incoming light $L_i(\,)$ w/ the the BRDF:

$$L_r(\underline{v}_r,\lambda) = \int \underbrace{L_i(\underline{v}_i,\lambda)}_{\text{light intensity}} \cdot \underbrace{f(\underline{v}_i,\underline{v}_r,\underline{n},\lambda)}_{\text{BRDF}} \underbrace{\langle \underline{v}_i,\underline{n}\rangle^+}_{\text{fore shortening factor}} \, d\underline{v}_i$$

If $\exists$ a finite no. of point light sources: $L_r(\underline{v}_r,\lambda) = \sum L_i(\lambda) \cdot f(\underline{v}_i,\underline{v}_r,\underline{n},\lambda)\cos\theta_i^+$



$\max(\cos\theta_i, 0)$.

★ Typically BRDF is split into a $\boxed{\text{DIFFUSE}}$ & $\boxed{\text{SPECULAR}}$ components

DIFFUSE Reflection (Lambertian) Matte surfaces (most textiles, paper, paints, wooden, vegetation, stone, concrete ...)

- Scatters light uniformly in all directions
- widely used, smooth / non-shiny surfaces

Intensity variety w.r.t. only the surface normal →

→ Now, BRDF is constant: $f(\underline{v}_i, \underline{v}_r, \underline{n}, \lambda) = f_d(\lambda)$: aka surface albedo

like glued to the surface

The shading equation for Diffuse Reflection model becomes

$f_d(P)$ op

$$L_d^{(P)}(\underline{v}_r, \lambda) = \sum_i L_i(\lambda) \cdot f_d(\lambda) \langle \underline{v}_i, \underline{n} \rangle^+$$

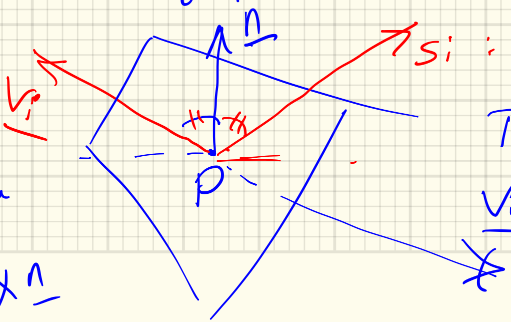
$f_d : \rho(P) : \mathbb{R}^3 \to \mathbb{R}^+$ typical of surface material

Scene Radiance at P in outgoing dir $\underline{v}_r$ (viewing dir. does not matter)

SPECULAR Reflection : Reflected (specular) direction is important.

mirror-like surfaces or glossy : metal, ceramic.

∴ Viewing direction (vantage point) $\underline{v}_r$ matters



s_i : coplanar w/ $(\underline{v}_i, \underline{n})$ plane : $\underline{v}_i$: light source direction

n: surface normal

Form a plane by v_i & $\underline{n}$

The amount of light reflected in a given direction $\underline{v}_r$ now depends on the angle btw $\langle \underline{v}_r, s_i \rangle$

→ Different variations exist, e.g. one BRDF for specular reflection

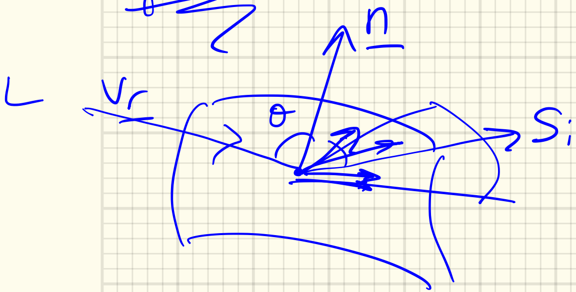
$$f_s(\underline{v}_r, \underline{s}_i, \lambda) = \underbrace{\langle \underline{v}_r, \underline{s}_i \rangle}_{(\cos\theta_s)^{k_e}}{}^{k_e} \cdot k_s(\lambda)$$

→ e.g. Phong model uses a power of the cosine

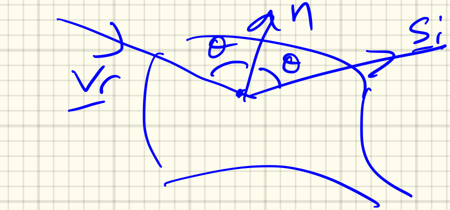
: larger k : more specular surface.

smaller k : materials w/ softer gloss.

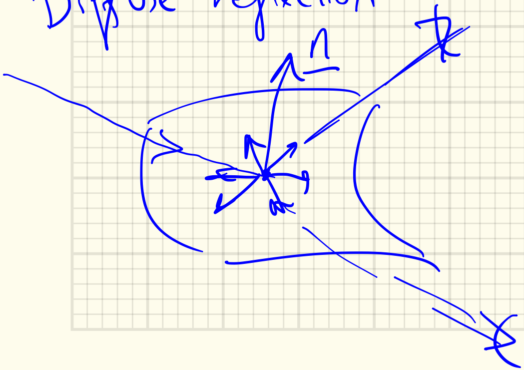
~~softer~~ glossy surfaces



~~strong~~ Specular surface



Diffuse reflection



Phong Shading Equation: Adds an ambient illumination term:

$$L_r(\underline{v}_r, \lambda) = \underbrace{k_a(\lambda)}_{\text{ambient color distrib}} L_a(\lambda) + \underbrace{k_d(\lambda)}_{\text{diffuse reflection}} \sum_i L_i(\lambda) \langle \underline{v}_i, \underline{n} \rangle^+$$

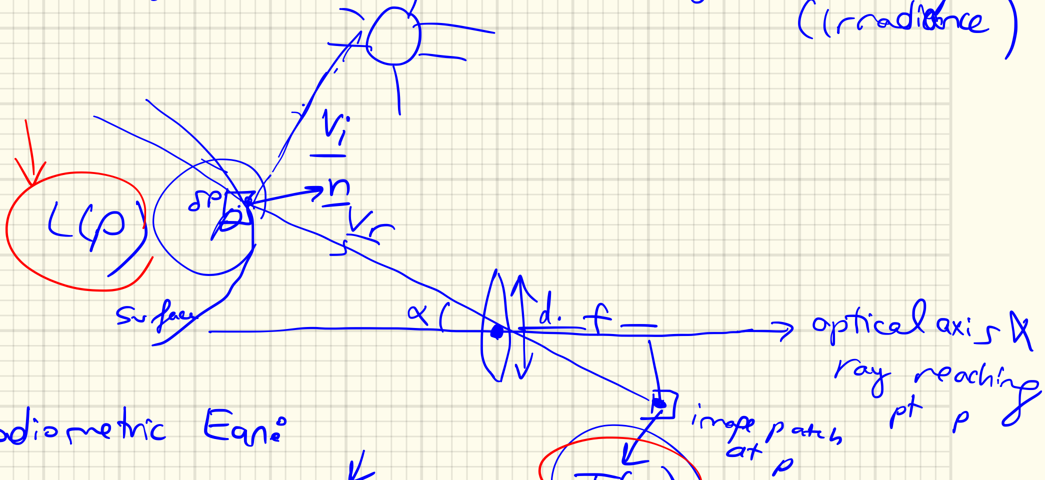
$$+ \underbrace{k_s(\lambda)}_{\text{Specular reflection}} \sum_i L_i(\lambda) \langle \underline{v}_r, \underline{s}_i \rangle^{k_e} \cdot (\underline{v}_i, \underline{n})^+$$

weights

eg. Sunny outdoor scene.
Ambient illumination $L_a(\lambda)$ may be blue.

eg. Indoor scene lit w/ candles : $L_a(\lambda)$ may be yellow.

⇒ Link Scene / Object Radiance to Image Intensity (Irradiance)



Fundamental Radiometric Eqn

$$I(p) \propto L(P)\,\underbrace{(\cos\alpha)^4}_{\angle(\underline{v_r},\ \text{optical axis})\ \text{foreshortening}} \cdot \left(\frac{d}{f}\right)^2$$

$$\frac{d}{f} \rightarrow \frac{1}{F_{\#}}$$

~~Thin lens~~

For a Pinhole camera

Irradiance Equation

$$\boxed{I(p) = \gamma\, L(P)}$$

$\alpha \searrow \cos\alpha \approx 1$

In basic radiometry : the simple model of Lambertian surface radiance is widely adopted :

* The fact that image intensity does not change w/ vantage point constitutes a FUNDAMENTAL condition to establish correspondence across multiple views (images) of the same object.