3D Vision

BLG634E Spring 2022

07.03.2022

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Recap

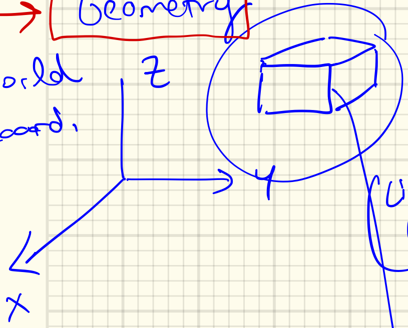
Image Formation:

① coord. xform

Projective Geometry

* → Geometry

World coord.



(R, T) → Rigid motion

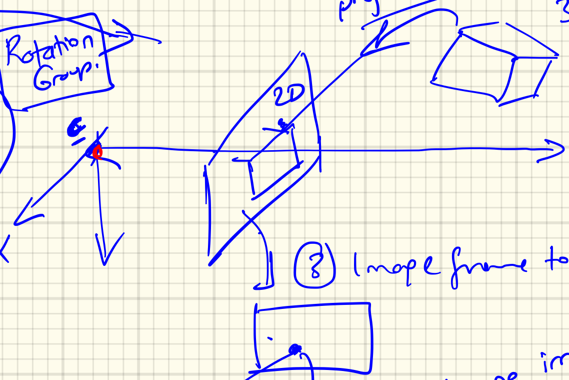
$SO(3)$ → Rotation Group.

Linear Algebra

Groups

projective transform ②

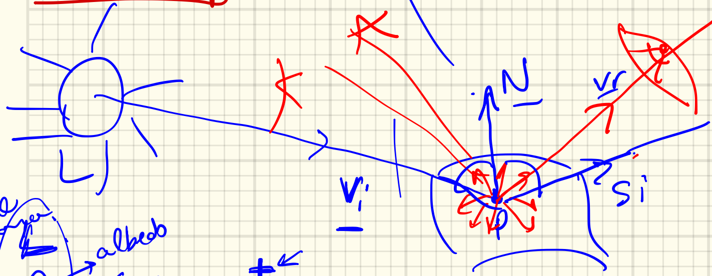
3D.



③ Image frame to pixel

$I(p)$: image irradiance

a little bit → Photometry



Scene radiance

$$k_d \; \rho \langle \underline{V_i}, \underline{N} \rangle_{(p)}^{+}$$

albedo

BRDF

Diffuse shading

Lambertian surface

Specular Shading term

Phong

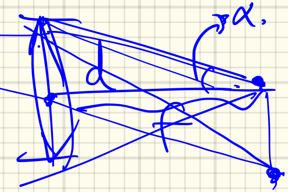
$$+ k_s \angle \langle \underline{V_r}, S_i \rangle^{k_e} \quad + k_a L_a$$

k_e ← heuristic exponents

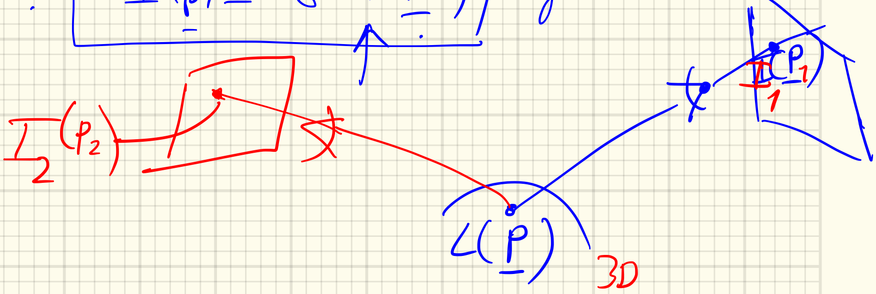
Scene Radiance.

[Trucco Verri 2.2]

$$I(p) = L(P)\underbrace{\left((\cos\alpha)^4\left(\frac{d}{f}\right)^2\right)}_{\gamma}$$



→ Image Irradiance Eqn: $\boxed{I(p) = \gamma L(P)}$ for a Lambertian



★ Establishing correspondences

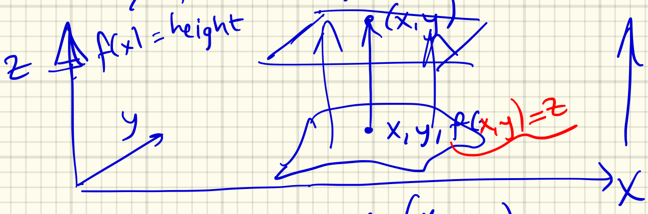
$$\boxed{I_1 \simeq I_2}$$

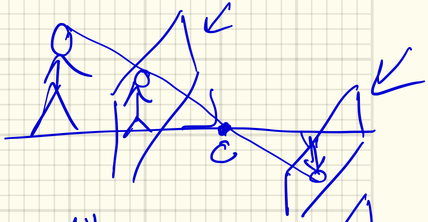
p_1 & p_2 on the images

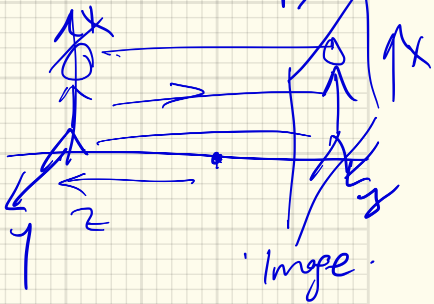
Photometric Stereo / Shape from Shading : Recover surface normals from images

Lambertian assumption $\longrightarrow$ Diffuse shading eqn.

Orthographic Projection.







$\underline{x} = (x, y, z)$

Shading eqn:

$$I(\underline{x}) = \rho(\underline{x})\,(\underline{N} \cdot \underline{V}) \cdot \underline{I}(\underline{x}, L)$$

$$I(x) = L_d(\underline{x}) = \rho \cdot N \cdot V_i$$

$$I(x) = (N^x, N^y, 1) \cdot \underline{V_i}$$

?

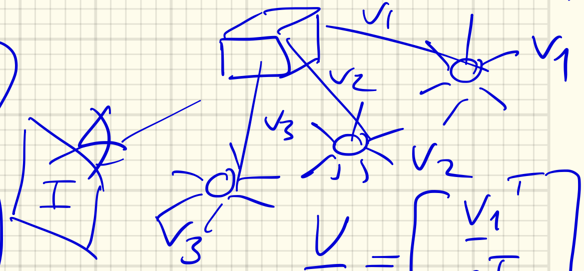
Basic Photometric Stereo:

Shape from Multiple Shaded Images (Forsyth 2.2.4)

Use orthographic projection:

$$I(x) = \rho(x) \cdot (N \cdot V_i)$$

$$= \underline{g}(x) \cdot \underline{V}_i$$



$$\underline{\underline{V}} = \begin{bmatrix} \underline{V}_1^T \\ \underline{V}_2^T \\ \vdots \\ V_n^T \end{bmatrix}$$

Use $\underline{V}_1, \ldots, \underline{V}_n$: known illuminant sources:

Measure image intensities at the same location:

$$\underline{\underline{I}}(x) = [\, I_1(x)\ I_2(x) \ldots I_n(x) \,]$$

$$\Rightarrow \underline{I}(x) = \underline{\underline{V}}\ \underline{g}(x) \longrightarrow \text{solve for } \underline{g}(x) \longrightarrow \underline{N}(x)$$

w/ Least Squares

From $\underline{N} = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, 1 \right) \longrightarrow$ e.g. by integrating solve for the surface $f(x,y) = z$.

Introductory Projective Geometry (HZ Chapter 1)

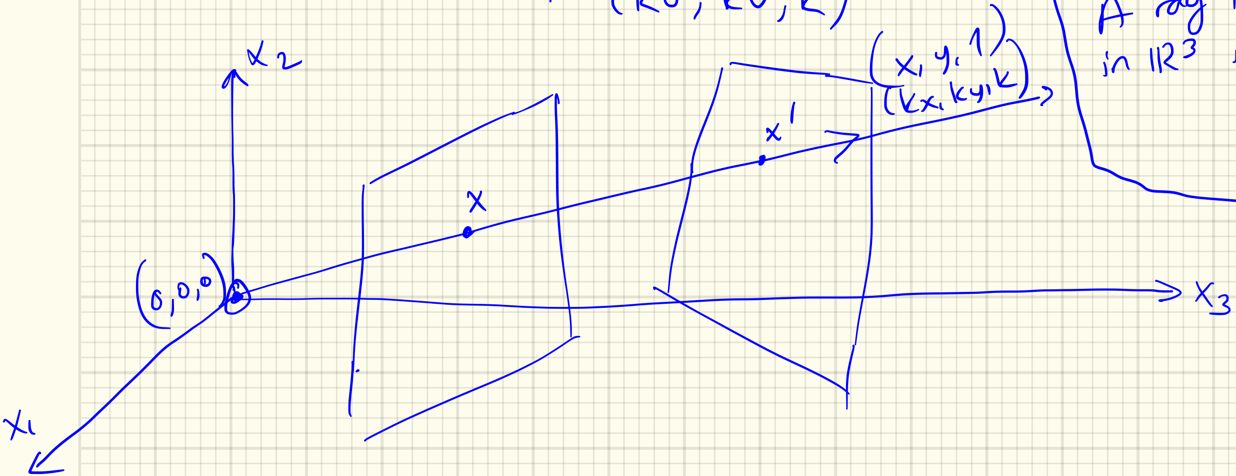
Projective spaces $\mathbb{P}^2$ & $\mathbb{P}^3$

$\mathbb{P}^2$: extension of plane $\mathbb{R}^2 \longrightarrow \mathbb{P}^2$.

$$\underset{\mathbb{R}^2}{(u,v)^T} \longrightarrow \underset{\mathbb{P}^2 \subset \mathbb{R}^3}{(u,v,1)^T}$$

$$\simeq (ku, kv, k)^T$$

Def: A ray thru the origin in $\mathbb{R}^3$ is a point in $\mathbb{P}^2$



Def: Equivalence class of vectors is known as a "homogeneous" vector

Def: The set of equivalence class of vectors in $\mathbb{R}^3 - (0,0,0)^T$ forms the Projective Space $\mathbb{P}^2$.

$$\mathbb{R}^2 \quad (x_1, x_2) \longrightarrow k(x_1, x_2, 1) \quad \mathbb{P}^2$$

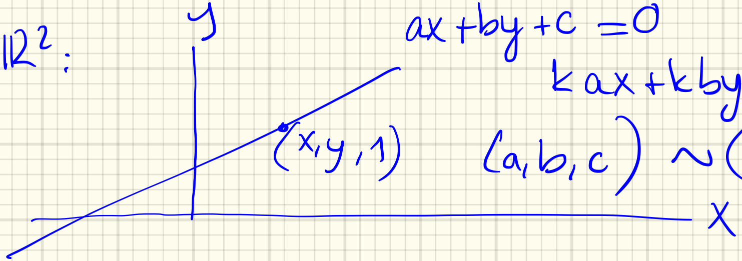
$$\mathbb{R}^n \quad (x_1, x_2, \dots, x_n) \longrightarrow \underbrace{k(x_1, x_2, \dots, x_n, 1)}_{(x_1, x_2, \dots, x_{n+1})} \in \mathbb{P}^n$$

$$\mathbb{R}^n \quad \left(\frac{x_1}{x_{n+1}}, \frac{x_2}{x_{n+1}}, \dots, \frac{x_n}{x_{n+1}}\right) \longleftarrow (x_1, x_2, \dots, x_{n+1})$$

* A line in the plane $\mathbb{R}^2$:

$$\underline{l} = (a, b, c)$$

$$\underbrace{(x, y, 1)}_{\underline{X}} \cdot \underline{l} = 0$$



$$ax + by + c = 0 \longrightarrow (a, b, c)$$

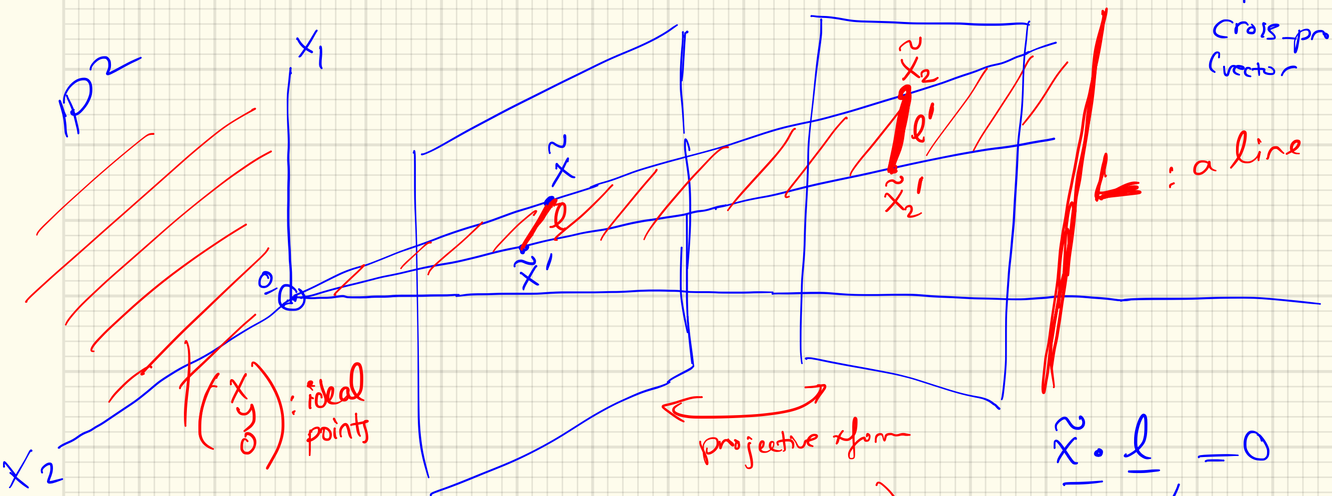
$$kax + kby + kc = 0$$

$$(a, b, c) \sim (ka, kb, kc)$$

Def: → homogeneous eqn of a line $\boxed{\underline{x} \cdot \underline{l} = 0}$: point $\underline{x}$ lies on a line $\underline{l}$

Def: Line joining two points $\tilde{x}, \tilde{x}'$: $\underline{l} = \tilde{x} \times \tilde{x}'$

cross product (vector)



$\underline{\tilde{x}} \cdot \underline{l} = 0$

$\tilde{x} \cdot (\tilde{x} \times \tilde{x}') = 0$ ✓

* In $\mathbb{P}^2$: points are rays thru the origin $\underline{O}$.
: lines are planes " " $\underline{O}$.

Def: Intersection of two lines $l = (a, b, c)^T$, $l' = (a', b', c')$

→ a point in $\mathbb{P}^2$ $\underline{x} = \underline{l} \times \underline{l}'$

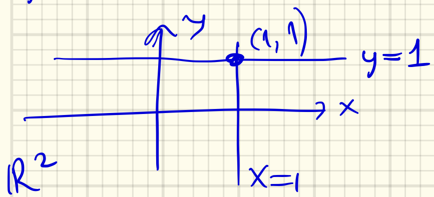
e.g. Determine intersection btw

$$\mathbb{R}^2$$

$$x = 1 \rightarrow 1 - x = 0 \rightarrow (-1, 0, 1) : \underline{l}_1$$

$$y = 1 \rightarrow 1 - y = 0 \rightarrow (0, -1, 1) : \underline{l}_2$$

homogeneous repres. of lines

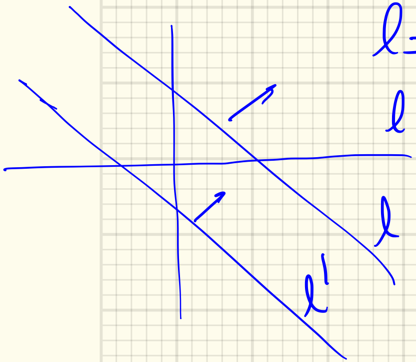


$$\underline{x} = \underline{l}_1 \times \underline{l}_2 = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{vmatrix} = \underbrace{(1, 1, 1)}_{\tilde{x}}$$

$$\Downarrow$$

$$x \rightarrow (1, 1) \quad \text{: inhomogenous pt coord.}$$

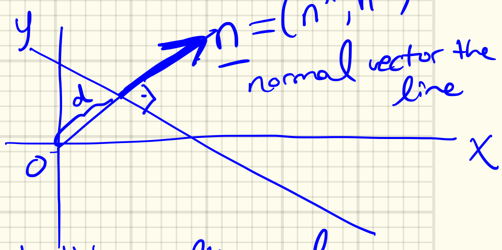
Intersection of Parallel lines:



$$\left.\begin{aligned} \underline{l} &= (a, b, c) \\ \underline{l}' &= (a, b, c') \end{aligned}\right\} \text{parallel}$$

$$\underbrace{\underline{l} \times \underline{l}'}_{\text{point of intersection}} = \underbrace{(c' - c)}_{k} \underbrace{(b, -a, 0)}$$

Note: $\mathbb{R}^2$



d: distance of the line l to the origin

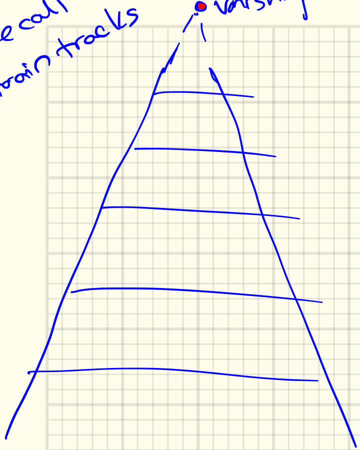
$$\underline{l} = (n^x, n^y, d) \quad \text{: homogeneous}$$

$$\underline{l} = (a, b, c) \sim (ka, kb, kc)$$

$$\underline{l} = \left(\frac{a}{\sqrt{a^2 + b^2}}, \frac{b}{\sqrt{a^2 + b^2}}, \frac{c}{\sqrt{a^2 + b^2}} \right)$$

Recall train tracks

vanishing pts.



In Homogeneous coord. $\to$ intersection point of 2 parallel lines

$$(b, -a, 0)$$

$$(x_1, x_2, 0)$$

Homogeneous coord: $\underline{x} = (x_1, x_2, x_3) \to x_3 \neq 0$ finite points

$\mathbb{P}^2$

$\underline{x} = (x_1, x_2, 0)$, $x_3 = 0$ ideal pt or a point at ∞.

Inhomogeneous coord: finite pt $\left(\frac{x_1}{x_3}, \frac{x_2}{x_3}\right)$

$\vdots \left(\frac{x_1}{0}, \frac{x_2}{0}\right)$!

Cannot be modeled in Euclidean geometry.

Def (In $\mathbb{P}^2$): Points w/ last coord $= 0$ are called Points at ∞ (Ideal Points)

point at ∞ is mapped to a finite point!

$$\underbrace{H}_{\text{projective xform}} \cdot \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \to \text{finite point!}$$

3D Points:

$\underline{x} = (x, y, z) \in \mathbb{R}^3 \longrightarrow \tilde{\underline{x}} = (\tilde{x}, \tilde{y}, \tilde{z}, \tilde{w}) \in \mathbb{P}^3$

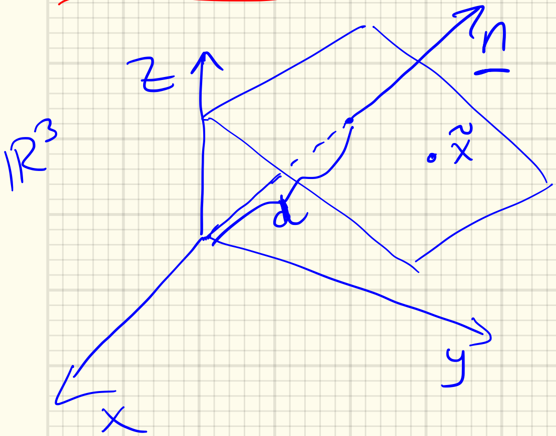
$\mathbb{P}^3 = \mathbb{R}^4 - (0,0,0,0)^T$.

$\tilde{x}$ is augmented x : $(x, y, z, 1)$

$k(x, y, z, 1)$ $\sim$

3D Planes:

$\underline{m} = (a, b, c, d)$: $\boxed{ax + by + cz + d = 0}$



$\tilde{x} = (x, y, z, 1)$

$\underline{m} \cdot \tilde{x} = 0 \leftarrow$

$\underline{m} = (\underbrace{n^x, n^y, n^z}_{\underline{n}}, \hat{d})$, $\|\underline{n}\| = 1$

$= \left(\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{}}, \frac{c}{\sqrt{}}, \frac{d}{\sqrt{}} \right)$

In Computer Vision, 2 important class of projective transformations : Linear xformations btw projective spaces

(1) Transformations btw $\mathbb{P}^3$ & $\mathbb{P}^2$ which models (image formation) projection.

$\mathbb{P}^3 \longrightarrow \mathbb{P}^2$

$$\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{3\times1} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{3\times4} \underbrace{\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}_{4\times1}$$

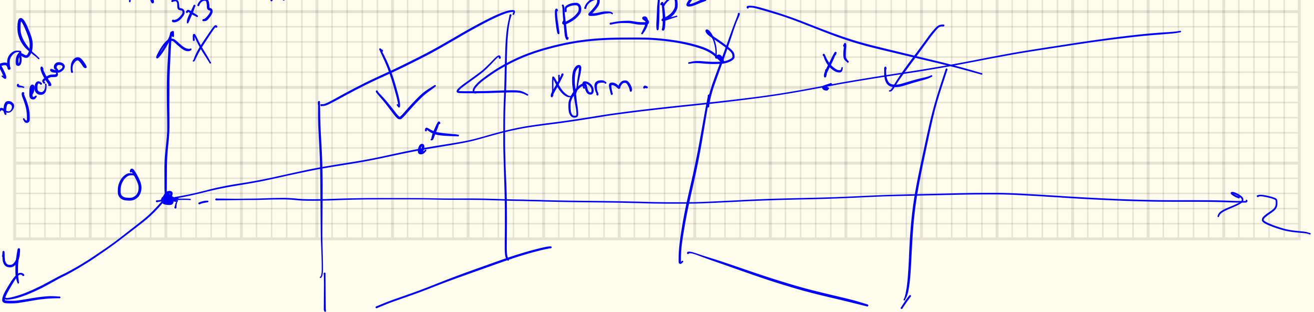
$$\mathbb{P}^2 \xleftarrow{\ \Pi_0\ } \mathbb{P}^3$$

$$\left(\frac{x}{z}, \frac{y}{z}\right)$$

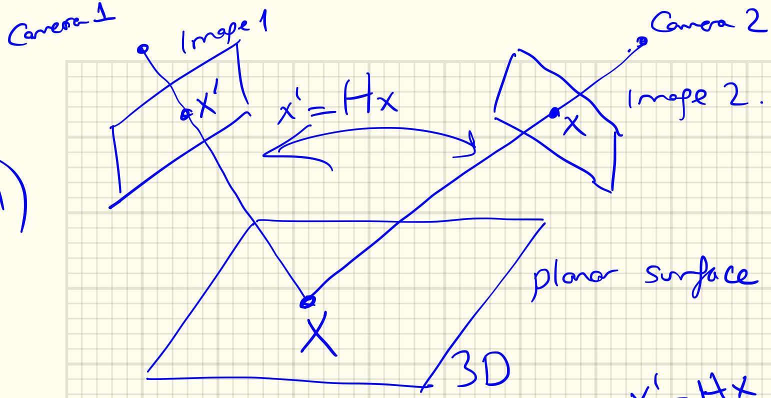
(2) Linear invertible transformations of $\mathbb{P}^n$ into themselves

$H_{3\times3} : \mathbb{P}^2 \longrightarrow \mathbb{P}^2$: Homographies

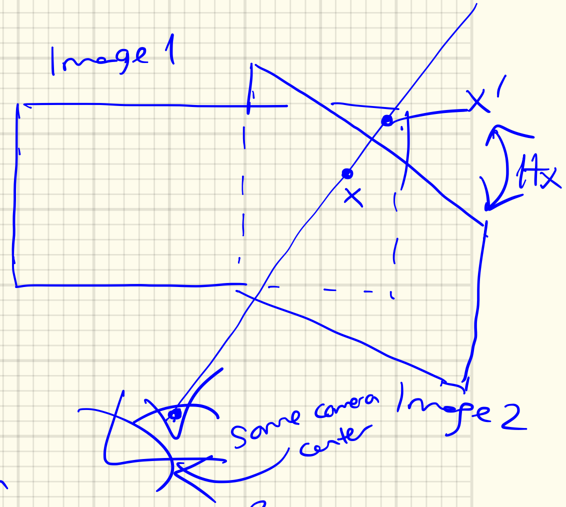
e.g. central projection



1)



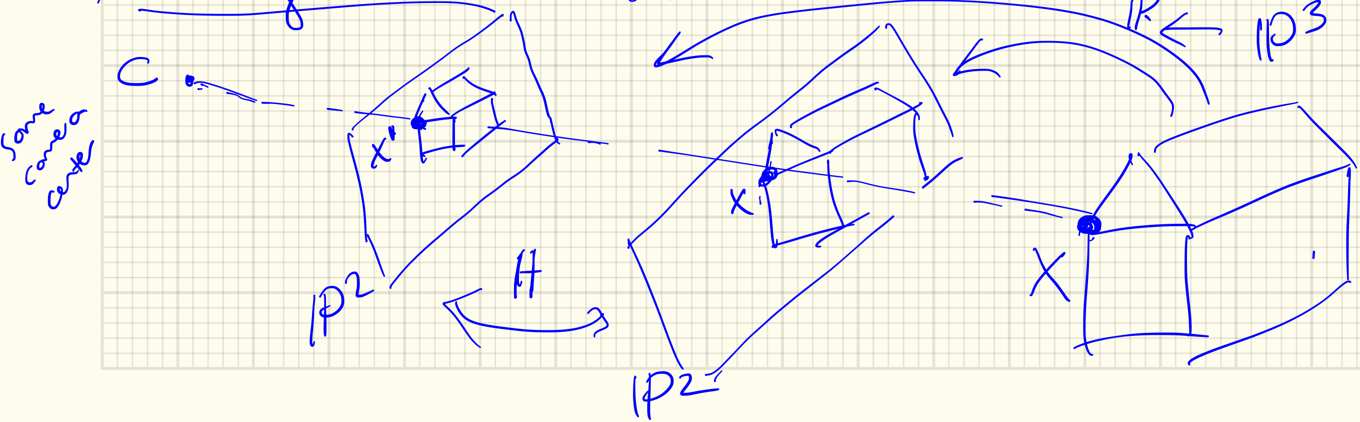
2) Rotating Camera:



$$x' = Hx$$

$$Kx' = K\,H\,X.$$

3) Zooming Camera: Changing focal length



Def (Projective Transform): A mapping $h: \mathbb{P}^2 \to \mathbb{P}^2$ is a projective xform iff $\exists$ a non-singular 3×3 matrix H s.t. for any point in $\mathbb{P}^2$: $\underline{x}' = \underline{\underline{H}}\,\underline{x}$

$\underline{\underline{H}} \in \underbrace{GL(3)}$

General Linear Group. ?

Rotation Group $SO(3)$.

Next time: Linear Algebra Groups

Reading: HZ Book Chapter 1.

homogeneous repres. of a ~~conic~~

$x^T \underline{\underline{C}}\, x = 0$

$$\begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

a, b, c, d, e

conic.