3D Vision

BLG634E

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Last time: 3D Rotations: $SO(3)$: R: 3x3 matrices: $R^T R = I$, $\det(R) = +1$

1) Euler Angles ✗

2) Exponential Coordinates: $R(t) = e^{\hat{\omega} t}$

amount of rotation around $\underline{\omega}$: t

$so(2)$

$\underline{\omega}$

$\underline{\omega} \xrightarrow{\wedge} \hat{\omega}$

$[\;] \qquad [\quad]$

Given a rotation axis $\underline{\omega}$, $\xrightarrow{\text{exp map}} R(t)$, $t = \|\omega\|$, t.

$\boxed{\text{exp map}}$ Rodrigues formula ✓ (exponential matrix formula)

$so(3) \rightarrow SO(3)$

Given a rotation matrix $R \longrightarrow \underline{\omega}$

log map $(SO(3)) \longrightarrow so(3)$ $\underline{\omega}$

3) Quaternions: used by Computer Graphics.

$\mathbb{H}: \mathbb{C} + j\,\mathbb{C}$

$k^2 = j^2 = i^2 = -1$

$k = i \cdot j$

$i \cdot j = -j \cdot i$

q_0

$\underline{v}$

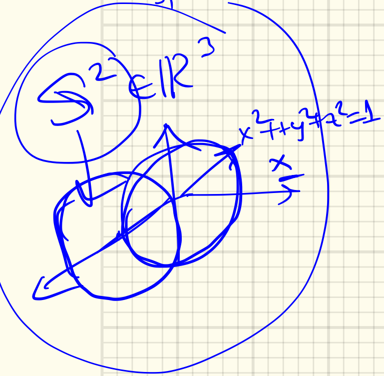
$\underline{q} \in \mathbb{H} = (q_0, \underline{v}) = q_0 + q_1 i + q_2 j + q_3 k$

$\in \mathbb{R}^4$

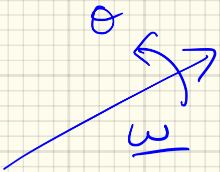
Quaternion multiplication: $q_1 * q_2$

$\rightarrow$ We embed the $SO(3)$ into $\mathbb{H}$ unit quaternion.

Unit quaternion space $= S^3 \in \mathbb{R}^4$ sphere in $= \{ \underline{q} \in \mathbb{H} : \|q\|^2 = q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \}$



exercise: Show that S^3 is a group, w/ quaternion multiplication



A rotation around axis $\underline{\omega}$ (a unit vector) by an angle θ:

$*$ The quaternion that computes this rotation: $\underline{q} = \left(\cos\frac{\theta}{2}, \underline{\omega}\left(\sin\frac{\theta}{2}\right)\right)$

$*$ A point $\underline{P}$ in space is represented by a quaternion $\rightarrow \underline{q}\, x_p$

$$\underline{p} = (0, \underline{P})$$

$\Rightarrow$ A desired rotation of the point $\underline{p}$ $\Rightarrow$ $\boxed{\underline{p}_{rotated} = \underline{q} * \underline{p} * \underline{q}^{-1}}$

quaternion multiplication

Recall: conjugate quaternion $\bar{q} = q_0 - q_1 i - q_2 j - q_3 k$

$$\underbrace{\underline{q}\,\bar{\underline{q}} = \|\underline{q}\|^2}_{} = q_0^2 + q_1^2 + q_2^2 + q_3^2$$

Inverse for $\underline{q}$: $\underline{q}^{-1} = \dfrac{\bar{q}}{\underbrace{\|q\|^2}_{=1}}$ $\Rightarrow$ in S^3 $(\|H\|)$

$$q^{-1} = \bar{q}$$

$\Rightarrow$ * Given a rotation matrix $R = e^{\hat{\omega} t}$ w/ $\|\omega\| = 1$, $t \in \mathbb{R}$, we can obtain a unit quaternion:

$$q(R) = \cos\left(\frac{t}{2}\right) + \sin\left(\frac{t}{2}\right)(\omega_1 i + \omega_2 j + \omega_3 k) \in S^3.$$

$\Rightarrow$ * Given a unit quaternion $\underline{q} = q_0 + q_1 i + q_2 j + q_3 k \Rightarrow R(q) = e^{\hat{\omega} t} \in SO(3)$

$$t = 2 \arccos(q_0), \quad \omega_m = \begin{cases} q_m / \sin(t/2), & t \neq 0 \\ 0 & t = 0 \end{cases}$$

$m = 1, 2, 3$

Quaternion ~~exp~~ coord. representation

→ Smooth parameterizations of the SO(3).

* Concatenating Rotations : Let q_1, q_2 be unit quaternions,

First apply $\underline{q_1}$ then $\underline{q_2}$:

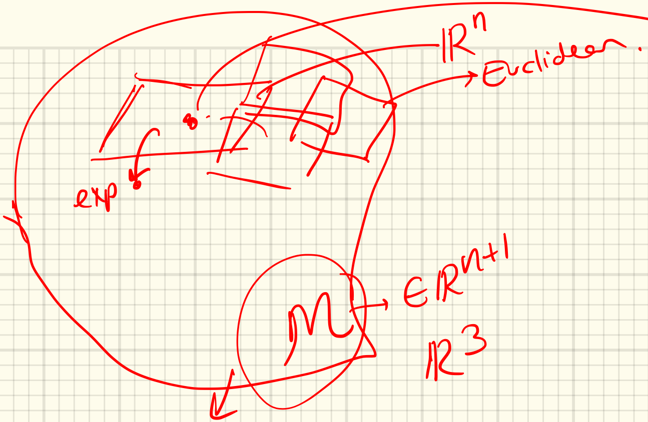
$$= q_2 * \left(q_1 * \underline{P} * q_1^{-1} \right) * q_2^{-1}$$

$$= (q_2 * q_1) * \underline{P} * \underbrace{(q_1^{-1} * q_2^{-1})}$$

$$= \underbrace{(q_2 * q_1)}_{\underline{q}} * \underset{\underline{P}}{\underline{P}} * \underbrace{(q_2 * q_1)^{-1}}_{\underline{q}^{-1}}$$

∴ Composite rotation is represented by the quaternion

$$\underline{q} = q_2 * q_1$$



M : 2D manifold

$$\mathfrak{so}(3) \xrightarrow{\text{exp map}} SO(3)$$

ξ

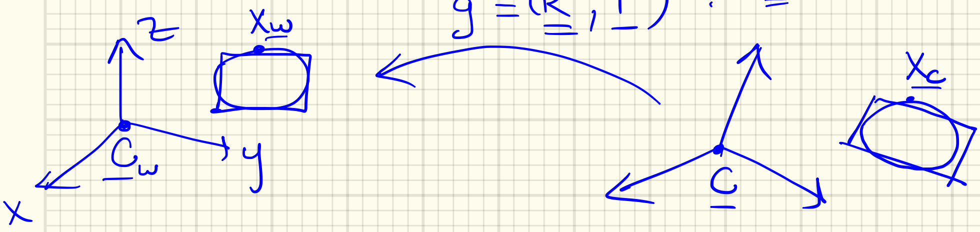
$\xleftarrow{\text{log map}}$

→ Differential Geometry

Riemannian Geometry.

Rigid Motion:

$g = (\underline{R}, \underline{T}) \in SE(3)$: $\underline{R} \in SO(3)$ (3D rotation), $\underline{T} \in \mathbb{R}^3$ (3D Translation)



$$\boxed{X_w = \underline{R}_{cw} X_c + \underline{T}}$$

Q: is this a linear map?

g: Transformation btw World & Camera Coord Frames.

Def $SE(3)$: space of rigid body motions

$$SE(3) = \{ g = (R, T) : \underline{R} \in SO(3), \underline{T} \in \mathbb{R}^3 \}$$

$X_c \overset{g}{\rightleftarrows} X_w$: not linear

→ **Homogeneous Representation of g:** Move g to a linear matrix representation using homogeneous coord.

$\underline{X} = (X_1, X_2, X_3) \in \mathbb{R}^3 \longrightarrow \underline{\tilde{X}} = (X_1, X_2, X_3, 1) \in \mathbb{R}^4$

Now: $\underline{\tilde{X}}_w = \begin{bmatrix} \underline{\underline{R}} & \underline{T} \\ \underline{0}^T & 1 \end{bmatrix}_{\underline{4\times4}} \tilde{X}_c = \tilde{g}\, \underline{X}_c$

Def: $\tilde{g} \in \mathbb{R}^{4\times4}$ is the homogeneous repres. of rigid body motion.

$$SE(3) \triangleq \left\{ \tilde{g} = \begin{bmatrix} \underline{\underline{R}} & \underline{T} \\ \underline{0}^T & 1 \end{bmatrix} \,\middle|\, R \in SO(3), T \in \mathbb{R}^3, \ \underline{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

Exercise: Show that $SE(3)$ is a group w/ this representation.

* $\tilde{g} \in SE(3)$: $\tilde{g}(\tilde{X}) = \underline{\underline{R}}\,\underline{X} + \underline{T}$ on a point

$\tilde{g}$ on a vector $\underline{v}$: $\tilde{g}\tilde{Y} - \tilde{g}\tilde{X} = \tilde{g}\,\underline{v} = \tilde{g}(\tilde{Y} - \tilde{X})$

$$= R\underline{Y} + T - (RX + T = R(Y - X) = \underline{\underline{R}} \cdot \underline{V}$$

$\therefore$ A vector is affected by rotational part of the rigid motion.

Hiearchy of Transformations in Homogeneous Representations

(1) Euclidean Transform:

SE(2): $$\underbrace{\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}}_{\underline{\tilde{x}}'} = \underbrace{\begin{bmatrix} \underline{R}_{2\times2} & \underline{t}_{2\times1} \\ \underline{0}^T_{1\times2} & 1 \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\underline{\tilde{x}}}$$

rigid-body

3 dof: θ: planar rot.

t_x, t_y: translation

min of 2 point correspondences to solve for SE(2) parameters.

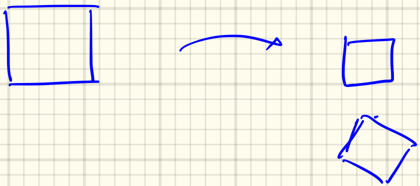
$$\underline{R} \in SO(2) = \{ R \in \mathcal{M}^{2\times2} : R^T R = I,\ \det R = +1 \}$$

(2) Similarity Transform:

Euclidean xform w/ isotropic scaling w/ scale s.

shape-preserving

$$\tilde{\underline{x}}' = \begin{bmatrix} s\underline{\underline{R}} & \underline{t} \\ \underline{0}^T & 1 \end{bmatrix}_{4\times4} \tilde{\underline{x}}$$



4 dof → min 2 pt correspondences.

Invariants: angles, ratios of lengths, ...

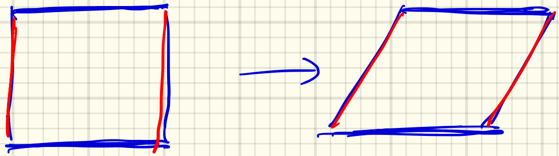
(3) Affine Transform:

$$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}} & \underline{t} \\ \underline{0}^T & 1 \end{bmatrix}_{3\times 3}$$

$\underline{\underline{A}} \in \overbrace{GL(2)}^{\text{set of all } 2\times 2 \text{ matrices that are invertible}}$

$t \in \mathbb{R}^2$

6 dof $\longrightarrow$ min 3 point correspondences



Invariants: parallel lines, ... ?

check from [HZ] book

4) Projective Transform:

$\underline{\underline{H}} \in GL(3) \setminus \mathbb{R}$ — defined upto a scaler

$$\underline{x}' = \underline{\underline{H}}\,\underline{x} \Rightarrow \underline{\underline{H}} = \begin{bmatrix} \underline{\underline{A}} & \underline{t} \\ \underline{v}^T & u \end{bmatrix}_{3\times 3} = \begin{bmatrix} [\underline{\underline{A}}]/u & \underline{t}/u \\ \underline{v}^T/u & 1 \end{bmatrix}_{3\times 3}$$

u: scalar

$\underline{\underline{H}}$: 8 dof : min 4 point correspondences between image planes.

(w/ no 3 points collinear on either plane)

Note:

Recall in $\mathbb{P}^2$

affine $\underline{\underline{A}}$

$$\underbrace{\begin{bmatrix} \underline{\underline{A}} & \underline{t} \\ \underline{0}^T & 1 \end{bmatrix}}_{} \cdot \underbrace{\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}}_{\text{point at } \infty} = \begin{bmatrix} x' \\ y' \\ 0 \end{bmatrix}$$

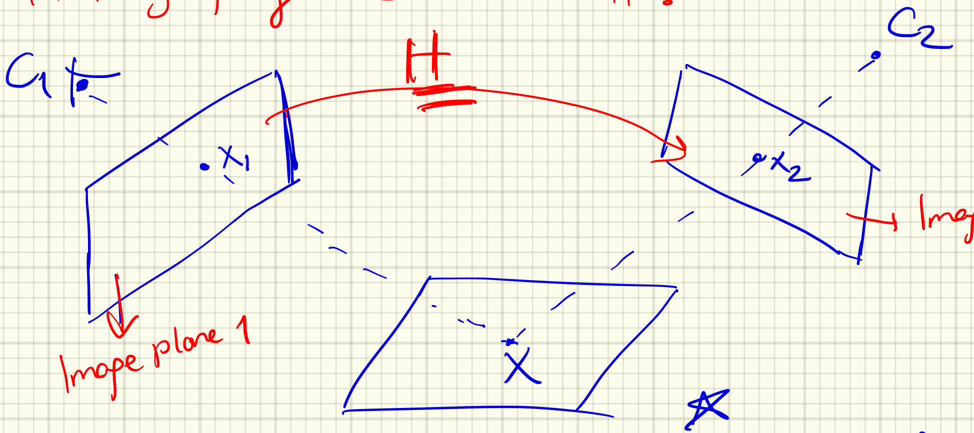
w/ $\underline{\underline{H}}$

$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \leftarrow \text{a finite point}$$

$w' \neq 0$

$\therefore \underline{\underline{H}}$ can model vanishing points.

Homography Estimation : (Projective transform = Homography)



$$x_2 = \underline{H}\, x_1 \quad ?$$

World Plane: A Planar surface in 3D imaged with 2 projective cameras <u>induce</u> a homography, between the 2 image planes.

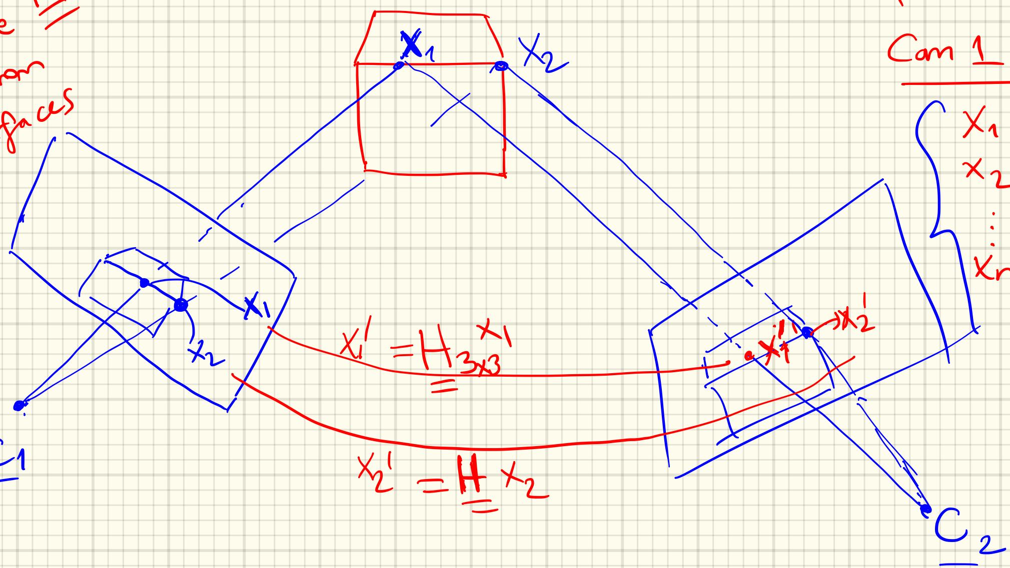
<u>Q</u> : How to obtain $\underline{H}$?

Homography : $\mathbb{P}^2 \to \mathbb{P}^2$

$\underline{\underline{H}}$

3D Object

* Points are selected from planar surfaces in 3D scene.



Given point corresp. from 2 images:

Cam 1		Cam 2
x_1	$\leftrightarrow$	x_1'
x_2	$\leftrightarrow$	x_2'
$\vdots$		
x_n	$\leftrightarrow$	x_n'

$\left\{ \underline{x}_i, \underline{x}_i' \right\}_{i=1}^{n}$

Problem: Compute the projective xform (homography), ie a 3x3 matrix $\underline{\underline{H}}$ s.t.

$$\underline{x}_i' = \underline{\underline{H}}\, \underline{x}_i \quad \forall i$$

homogeneous coord

8 dof.

→ Correspondence estimation is not a perfect process → introduces errors into the homography estimation.

8 dof * If 4 point correspondences are given, in theory an exact soln for matrix $\underline{\underline{H}}$ is possible.

* But points are measured inexactly → noise.

we need more than 4 point corresp → however they may not be fully compatible w/ a single projective transform.

Goal ⟹ Determine the "best" transform given the data

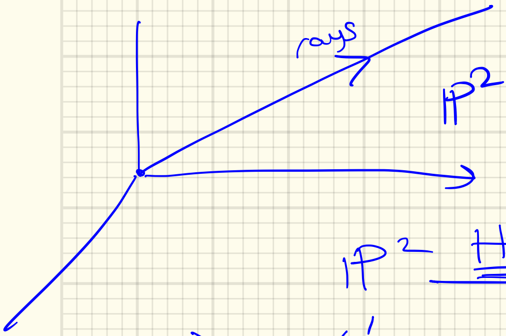
→ Find $\underline{\underline{H}}$ that minimizes some cost fn.

Direct Linear Transform (DLT) [Algorithm 3.1] [HZ 3.1]

$$\boxed{\underline{x}_i' = \underline{\underline{H}}\,\underline{x}_i}\ , \quad i = 1, \dots, n.$$

homogeneous points

correspondences



$\mathbb{P}^2$

$$\underline{x}_i \to \begin{pmatrix} wx \\ wy \\ w \end{pmatrix} \qquad \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\mathbb{P}^2 \xrightarrow{\underline{\underline{H}}} \mathbb{P}^2$$

$\Rightarrow$ $\underline{x}_i'$ & $\underline{\underline{H}}\,\underline{x}_i$ have the same direction.

$$\Rightarrow \quad \underline{x}_i' \otimes \underline{\underline{H}}\,\underline{x}_i = 0$$

cross product (vector)

$$\underline{x}_i' = \begin{pmatrix} x_i' \\ y_i' \\ w_i' \end{pmatrix} \qquad \underline{\underline{H}} = \begin{bmatrix} \underline{h}_1^T \\ \underline{h}_2^T \\ \underline{h}_3^T \end{bmatrix} \Rightarrow \underline{h}_j^T : j\text{th row of matrix } \underline{\underline{H}}.$$

$$\underline{\underline{H}}\,\underline{x}_i' = \begin{pmatrix} \underline{h}_1^T \underline{x}_i \\ \underline{h}_2^T \underline{x}_i \\ \underline{h}_3^T \underline{x}_i \end{pmatrix} \Longrightarrow \underline{x}_i' \otimes \underline{\underline{H}}\,\underline{x}_i = \underline{0}$$

$$\boxed{\underline{\underline{\hat{x}_i}} \cdot (\underline{\underline{H}}\,\underline{x}_i) = \underline{0}}$$

Recall hat operator map $x_i' \rightarrow$

$$\begin{bmatrix} 0 & -w_i' & y_i' \\ w_i' & 0 & -x_i' \\ -y_i' & x_i' & 0 \end{bmatrix} \underline{\underline{H}}\,\underline{x}_i = \underbrace{\begin{bmatrix} -w_i' \underline{h}_2^T \underline{x}_i + y_i' \underline{h}_3^T \underline{x}_i \\ w_i' \underline{h}_1^T \underline{x}_i - x_i' \underline{h}_3^T \underline{x}_i \\ -y_i' \underline{h}_1^T \underline{x}_i + x_i' \underline{h}_2^T \underline{x}_i \end{bmatrix}}_{3\times1} = \underline{0}$$

Re-arrange

$$\underbrace{\begin{bmatrix} \underline{0}_{1\times3} & -w_i' \underline{x}_i^T & y_i' \underline{x}_i^T \\ +w_i' \underline{x}_i^T & \underline{0} & -x_i' \underline{x}_i^T \\ -y_i' \underline{x}_i^T & x_i' \underline{x}_i^T & \underline{0} \end{bmatrix}}_{3\times9} \underbrace{\begin{bmatrix} \underline{h}_1 \\ \underline{h}_2 \\ \underline{h}_3 \end{bmatrix}}_{9\times1} = \underline{0} \qquad (\star)$$

$$\underline{\underline{A_i}} \cdot \underline{h} = \underline{0}$$

$(\underline{x}_i, \underline{x}_i')$ ↳ gave 2 eqns.

Check HZ book

* 3 eqns in ($\star$) but only 2 are linearly independent.
(notice 3^{rd} row $= x_i' \cdot 1^{st}$ row $+ y_i' \cdot 2^{nd}$ row)

* Each point correspondence gives 2 eqns in entries of $\underline{\underline{H}}$.

Solving for $\underline{\underline{H}}$: Given a set of n ($n \geq 4$) point correspondences,
4 or more

we obtain a set of equations:

$\underline{\underline{A}}$ is built from 1^{st} & 2^{nd} rows of $\underline{\underline{A}}_i$ matrices (on the prev pgs)

$$\boxed{\underline{\underline{A}} \cdot \underline{h} = 0}$$

$\underline{\underline{A}}_{2n \times 9}$ $\underline{h}_{9 \times 1}$

Homogeneous Overdetermined System of equations.

- $\underline{h}$ can be determined up to a scale.
- A scale is arbitrarily chosen s.t. $\|\underline{h}\|^2 = 1$.

We'll make use of SVD:

First a digression on SV $\Longrightarrow$

SVD: Any matrix $A_{m\times n}$: (Singular Value Decomposition)

$$\underline{\underline{A}}_{m\times n} = \underline{\underline{U}}_{m\times m}\, \underline{\underline{D}}_{m\times n}\, V^T_{n\times n}, \quad \underline{\underline{D}} = \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \\ 0 & \cdots & 0 \end{bmatrix}$$

— $\underline{\underline{D}}$: diagonal matrix of singular values σ_i

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0.$$

— The columns of $\underline{\underline{U}}$ ($\underline{\underline{V}}$) are mutually orthogonal unit vectors

Properties: (1) $\underset{\text{condition }\#}{C} = \dfrac{\sigma_1}{\sigma_n}$ tells us for $\underline{\underline{A}}$ how close it is to be a singular matrix (non-invertible)

$\underline{\underline{A}}$ is nonsingular iff $\forall \sigma_i > 0$.

2) $\#$ nonzero σ_i = rank (A)

3) $\underline{\underline{A}}$ is square : $A^{-1} = \underline{\underline{V}}\,\underline{\underline{D}}^{-1}\,\underline{\underline{U}}^T$

Even for a singular matrix

$$\tilde{D}^{-1} = \begin{bmatrix} 1/\sigma_1 & & & & 0 & \\ & 1/\sigma_2 & & & & \\ & & \ddots & & & \\ & 0 & & & 1/\sigma_k & 0 \\ 0 & & \cdots & & 0 & 0 \; \cdots \end{bmatrix}$$

$$\Rightarrow \underset{\text{pseudo inverse}}{\underline{\underline{A}}^{+}} = \underline{\underline{V}}\,\underline{\underline{\tilde{D}}}^{-1}\,\underline{\underline{U}}^T$$

4) Columns of $\underline{\underline{U}}$ corresponding to nonzero singular values span the range (A).

$\Rightarrow$ ☆ Columns of $\underline{\underline{V}}$ corresp. to zero singular values span the $\mathcal{N}ull\,(\underline{\underline{A}}) = \text{Kernel}\,(\underline{\underline{A}})$

5) $\left.\begin{matrix}\underline{\underline{A}}^T\underline{\underline{A}} \\ \underline{\underline{A}}\,\underline{\underline{A}}^T\end{matrix}\right\}$ eigen values of these matrices $\quad \lambda_i = \sigma_i^2 \quad$ for $\sigma_i \neq 0$.

$$\underline{\underline{A}}\,\underline{h} = \underline{0}$$ homogeneous system of equations.

To solve this, let's set up an optimization problem.

$$\underset{\underline{\underline{h}}}{\arg\min} \|\underline{\underline{A}}\,\underline{h}\|^2 \quad \text{s.t.} \quad \|h\|^2 = \underline{1}$$

constrained optimization problem

$\Rightarrow$ convert to unconstrained; $\rightarrow$ set up a Lagrange multiplier λ $\rightarrow$ eqn.

$$\mathcal{L}(\underline{h}) = (\underline{\underline{A}}\,\underline{h})^T(\underline{\underline{A}}h) - \lambda\underbrace{(\underline{h}^T\underline{h} - 1)}_{\rightarrow \text{constraint}}$$

$$\mathcal{L}(\underline{h}, \lambda) = \underline{h}^T \underline{\underline{A}}^T \underline{A}\,\underline{h} - \lambda(h^T h - 1)$$

take deriv. $\downarrow$ to minimize $\mathcal{L}$

$$\frac{\partial \mathcal{L}}{\partial \underline{h}} = 2\,\underline{\underline{A}}^T\underline{\underline{A}}\,h - 2\lambda\underline{h} = 0$$

$$\Rightarrow \boxed{\underline{\underline{A}}^T\underline{\underline{A}}\,\underline{h} = \lambda\,\underline{h}} \rightarrow$$

$$\underline{(A^T A)\,\underline{h} = \lambda \underline{h}} \rightarrow \text{eigenvalue eqn for } (\underline{A}^T \underline{A})$$

$\underline{h}$ is an evector
λ is an evalue

which evalue?

Insert this eqn into $\mathcal{L}(\underline{h}, \lambda)$:

$$\mathcal{L}(\underline{h}, \lambda) = \cancel{h^T \lambda h} - \cancel{\lambda h^T h} + \lambda = \lambda$$

minimize.

minimum of $\mathcal{L}(\underline{h}, \lambda)$ is reached at $\lambda = 0$, $\therefore$ the least eigenvalue of $\underline{\underline{A}}^T \underline{\underline{A}}$.

$\Rightarrow$ The solution to $\boxed{\underline{\underline{A}}\,\underline{h} = 0}$ homogeneous system of eqns problem $\xrightarrow{is}$ the unit eigenvector of $\underline{\underline{A}}^T \underline{\underline{A}}$ w/ the least eigenvalue.

$\rightarrow \underline{h} \longrightarrow \underline{\underline{H}}$ ✓ $\quad \|h\| = 1.$

Recall:

$\underline{\underline{A}}\,\underline{x} = \underline{b}$

inhomogeneous system eqn.

$\min \|Ax - b\|^2$

$\underline{X} = (A^T A)^{-1} A^T \underline{b}$

→ **Normalized DLT** (Algorithm 3.2 [HZ Book]) Read.

$\{\underline{x}_i\}_{i=1}^{n} \longleftrightarrow \{\underline{x}_i'\}_{i=1}^{n}$

$\{\underline{x}_i\}_{i=1}^{n}$ from Image 1; $\{\underline{x}_i'\}_{i=1}^{n}$ from Image 2 } separately transform these set of points so that

$\{\underline{x}_i\}$, $\{\underline{x}_i'\}$ have mean $\begin{pmatrix}0\\0\\1\end{pmatrix}$ & their average dist from origin $= \sqrt{2}$, ie. the average point shoulde $\begin{pmatrix}1\\1\\1\end{pmatrix}$

ie. $\boxed{\underline{\tilde{x}}_i = \underline{T}\,\underline{x}_i}$ (★)

$\underline{x}_i \xrightarrow{T} \underline{\tilde{x}}_i$

$\underline{x}_i' \xrightarrow{T'} \underline{\tilde{x}}_i'$

$$\underline{\underline{T}} = \begin{bmatrix} s & 0 & -t_x \\ 0 & s & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

similarity transform

$\boxed{\text{Algo } 3.2}$ for Homography Estimation : Input : Given $n \geq 4$ $2D \leftrightarrow 2D$

$\{x_i\} \longleftrightarrow \{x_i'\}$

Output : $\underline{\underline{H}}$ s.t. $\underline{x}_i' = \underline{\underline{H}}\, \underline{x}_i$.

(i) Normalize the points by (★) thru similarity xforms $\underline{\underline{T}}$ & $\underline{\underline{T}}'$ for both sets $\{x_i\}, \{x_i'\}$ separately.

(ii) Apply DLT (Algo 3.1) to obtain $\tilde{\underline{\underline{H}}}$ ←

(iii) Denormalize : $\tilde{x}_i' = \tilde{\underline{\underline{H}}}\, \tilde{x}_i$

$x_i' = \underline{\underline{H}}\, x_i$

original eqn.

$$\underline{\underline{T}}'\underline{x}_i' = \tilde{\underline{\underline{H}}}\,(\underline{\underline{T}}\,\underline{x}_i)$$

$$x_i' = \underbrace{(\underline{\underline{T}}')^{-1}\,\tilde{\underline{\underline{H}}}\,\underline{\underline{T}}}_{=\underline{\underline{H}}}\; x_i$$

$$x_i' = \underline{\underline{H}}\, x_i$$

$$\boxed{\underline{\underline{H}} = (\underline{\underline{T}}')^{-1}\,\tilde{\underline{\underline{H}}}\,\underline{\underline{T}}}$$

→ Robust Estimation through Random Sample Consensus (RANSAC)

Until now, given $\{x_i\}_{i=1}^{N} \longleftrightarrow \{x_i'\}_{i=1}^{N}$

only source of error was assumed to be in the measurement of points (w/ a Gaussian distribution)

☆ However, points can be mismatched! → OUTLIERS

→ Outliers severely disturb the estimated homography.

$\boxed{\text{Idea}}$: RANSAC: Determine the inliers from correspondences

→ then homography estimation is robust.

Normalized DLT w/ RANSAC :

measure the error:

$$e = \| x_i' - \underline{\underline{H}}_{est} x_i \|^2 < \varepsilon$$

threshold.

Algo: Homography Estimation w/ RANSAC:

Reading Assignment Chapter 3 [HZ] 3.7

1) Normalize the point sets $\{x_i\}_{i=1}^{n} \leftrightarrow \{x_i'\}_{i=1}^{n}$

2) Pick $\underbrace{\text{min. \# required points}}_{=4}$ to estimate $\underline{\underline{H}}$

∃ rule of thumbs for parameter selection

3) Use Normalized DLT to estimate $\underline{\underline{\tilde{H}}}$

4) Denormalize to get $\underline{\underline{H}}$.

5) $\forall i, \; e_i = \| \underline{x}_i' - \underline{\underline{H}}\,\underline{x}_i \|^2 < \varepsilon$ ⇐ hyper parameter to choose

Count the inliers & note.

Go to (2)

Iterate this m times (random sampling) m: a parameter

→ Pick the iteration w/ the max # inliers.

→ Recalculate $\underline{\underline{H}}$ w/ all the inlier points (not just with the min # points = 4 pts here)