3D Vision

BLG634E

06.04.2021

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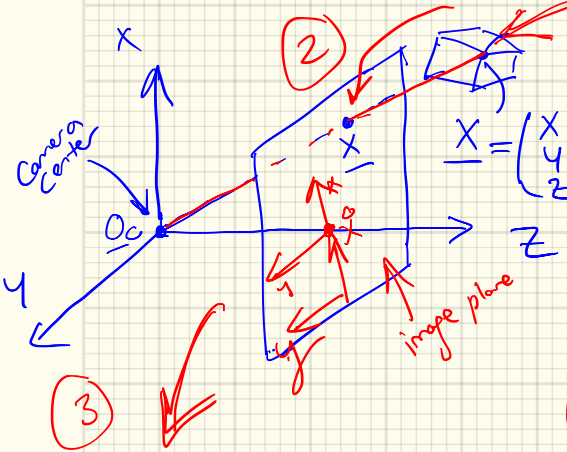
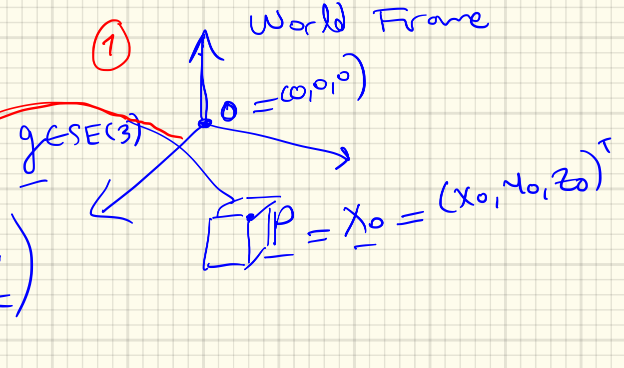
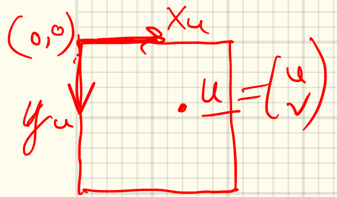
28.03.2022

Camera Parameters







1. World to Camera Frame Coord. Xform
2. Projection of 3D Coordinates into 2D image Coord
3. Coordinate transform btw 2D metric (normalized) image coordinates $\underline{x}$ and 2D pixel coord. $\underline{u}$.

⓪ Move to homogeneous coord.

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \rightarrow \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix}$$

①

$$\underline{X}_0 \rightarrow \underline{X} = \bar{g}\, \underline{X}_0$$

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \underline{\underline{R}}_{3\times 3} & \underline{T}_{3\times 1} \\ \underline{0}^T_{1\times 3} & 1 \end{bmatrix}_{4\times 4}}_{\bar{g} \in SE(3) \rightarrow (\underline{\underline{R}}, \underline{T})} \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{bmatrix}$$

$$: \quad \underline{\underline{R}} \in SO(3), \quad \underline{T} \in \mathbb{R}^3$$

3D World to Camera Transformation

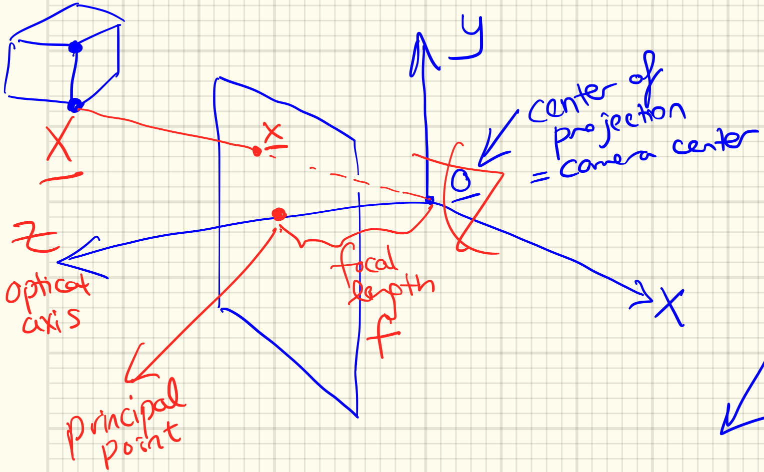
② Adapting pinhole camera model: Project $\underline{X}$ onto the image plane at the point $\underline{x} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$.

Recall perspective projection

$$\underline{x} = \begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{Z}\begin{bmatrix} X \\ Y \end{bmatrix}$$

↳ convert to homogeneous coord.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} fX \\ fY \\ Z \end{bmatrix}$$



Z: depth, unknown

write Z as as a scalar

$$Z\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3\times4}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \underbrace{\lambda \, \underline{x}}_{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}} = \underbrace{\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\underline{\underline{K}}_f \in \mathbb{R}^{3\times 3}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\underline{\underline{\Pi}}_0 \in \mathbb{R}^{3\times 4}} \underline{X} \rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

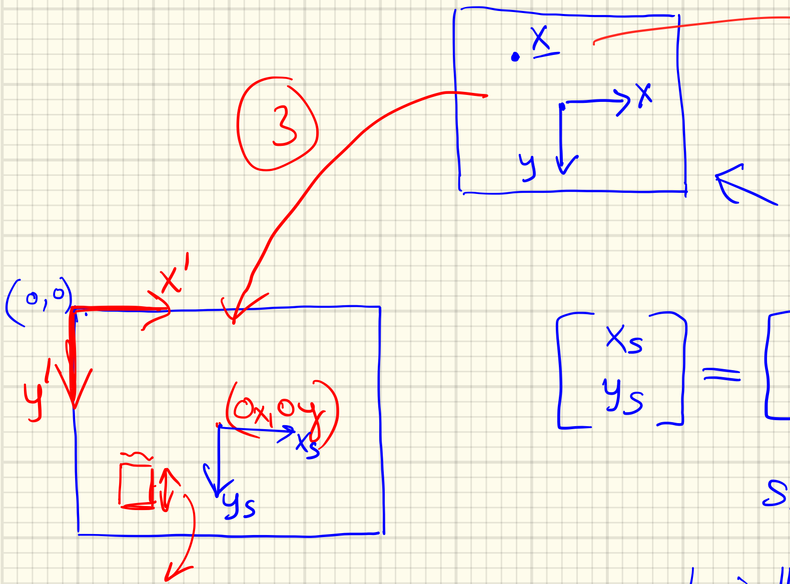
Π_0 : standard (canonical) projection matrix

$\mathbb{P}^3 \rightarrow \mathbb{P}^2$

To summarize :

$$\lambda \, \underline{x} = \underline{\underline{K}}_f \, \underline{\underline{\Pi}}_0 \, \underline{X} = \underline{\underline{K}}_f \, \underline{\underline{\Pi}}_0 \, \bar{g} \, \underline{X}_0$$

3. W/ a digital camera, measurements are typically provided in pixels w/ origin of image coordinate frame typically in the upper left corner of the image.



Metric Coordinates

(Normalized image coordinates)

$$\begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$S_x, S_y \propto \frac{1}{mm}$, relate to size of pixels

typically $S_x = S_y$

$\begin{bmatrix} o_x \\ o_y \end{bmatrix}$: principal point

Translate the origin to upper left corner of the image.

$$x' = x_s + o_x$$
$$y' = y_s + o_y$$

(o_x, o_y) : coord. in pixels of the principal point on the image plane.

Now all pixel coordinates are POSITIVE.

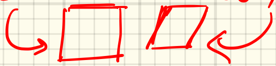
Actual (digital) image coord

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} s_x & \boxed{s_\theta} & o_x \\ 0 & s_y & o_y \\ 0 & 0 & 1 \end{bmatrix}}_{K_s \in \mathbb{R}^{3\times 3}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

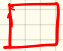
another linear transform

$\boxed{s_\theta : \text{skew factor}}$

$s_\theta = 0$ $\quad$ $s_\theta \neq 0$



When $s_x = s_y$



square

$s_\theta \neq 0$ when pixels are not rectangular

Usually $s_\theta = 0$
(assumed)

Now combine all the models

$$\lambda \underbrace{\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}}_{\text{pixel coordinates}} = \underbrace{\underbrace{\begin{bmatrix} s_x & s_\theta & o_x \\ 0 & s_y & o_y \\ 0 & 0 & 1 \end{bmatrix}}_{\underline{\underline{K}}_s} \underbrace{\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\underline{\underline{K}}_f}} \underbrace{\begin{bmatrix} \underline{\underline{I}}_{3\times 3} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix}}_{\underline{\underline{\Pi}}^0_{3\times 4}} \underbrace{\begin{bmatrix} \underline{\underline{R}} & \underline{T} \\ \underline{0}^T & 1 \end{bmatrix}}_{\overline{g}_{4\times 4}} \underbrace{\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix}}_{\text{world point}}$$

$$\underline{\underline{K}} \triangleq \underline{\underline{K}}_s \underline{\underline{K}}_f$$

Π_0 : perspective Projection matric

$\underline{\underline{K}}$: intrinsic (internal) camera matrix :

depends on $\boxed{\text{intrinsic camera parameters}}$:

f : focal length

s_x, s_y, s_θ : scaling factors $\left(\frac{1}{mm}\right)$ & a skew $\longrightarrow \sim$

o_x, o_y : center offsets

$\overline{g}$: extrinsic camera parameters (R, T) (external)

$$\underline{\underline{K}} = \begin{bmatrix} f s_x & f s_\theta & o_x \\ 0 & f s_y & o_y \\ 0 & 0 & 1 \end{bmatrix}, \quad \lambda \underline{x}' = \underline{\underline{K}} \, \underline{\underline{\Pi}}_0 \, \underline{X}$$

$\underline{\underline{K}}$: upper triangular matrix : When the calibration matrix $\underline{\underline{K}}$ is known, the calibrated coord. $\underline{x}$ can be obtained from $\underline{x}'$ by:

$$\lambda \underline{x} = \lambda \underline{\underline{K}}^{-1} \underline{x}'$$

The problem of estimating $\underline{\underline{K}}$'s parameters is called intrinsic camera calibration.

Extrinsic Camera Calibration: Estimate $\bar{g}$ parameters

Defines geometric relation between the world point $\underline{X}_0$ & camera point $\underline{X}$.

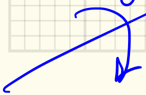
The overall model: $\lambda \underline{x}' = \underline{\underline{K}}\,\underline{\underline{\Pi}}_0\,\underline{X} = \underline{\underline{K}}\,\underline{\Pi}_0\,\bar{g}\,\underline{X}_0$
for pinhole camera projection

$$\lambda \underline{x'} = \underline{\underline{K}}\,\underline{\Pi}_0\,\bar{g}\,\underline{X}_0$$

Camera Calibration: 11 parameters to be estimated.

Intrinsics: $f\,,\ s_x,\ s_y\,,\ o_x,\ o_y$ $\quad (s_\theta) \quad \rightarrow 5$

Extrinsics: $\underline{\underline{R}}\,,\ \underline{\underline{T}} \quad \rightarrow 6$

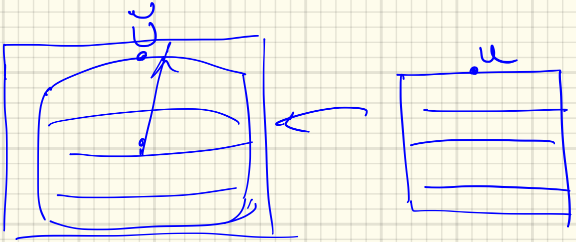


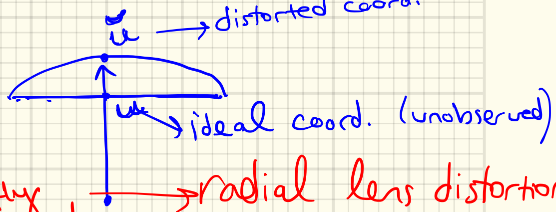
$\theta \rightarrow \underline{r} \leftarrow \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \rightarrow 3 \qquad \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$

④ Lens Distortion Camera Parameters :

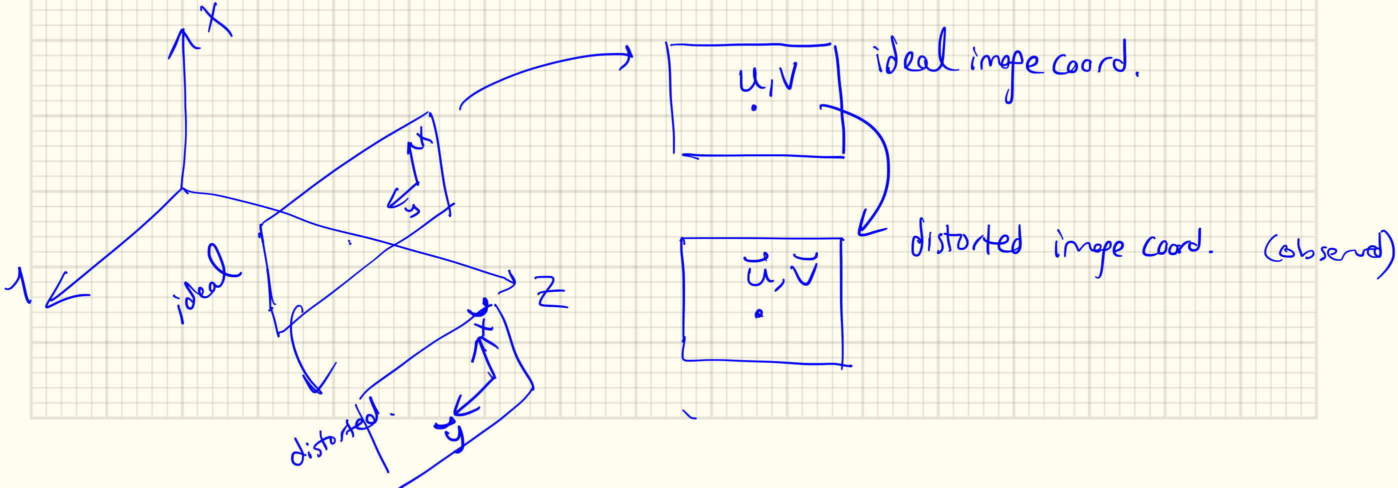
Optics of the lens introduces image distortions that show straight lines as curved lines.

effect





mostly adopted → radial lens distortion model :



$$\breve{x} = x\left(1 + k_1 r^2 + k_2 r^4\right) \qquad r^2 = x^2 + y^2$$

$$\breve{y} = y\left(1 + k_1 r^2 + k_2 r^4\right)$$

$\Rightarrow k_1, k_2$: (Radial) Lens Distortion Parameters

Write it in (u,v) coord.

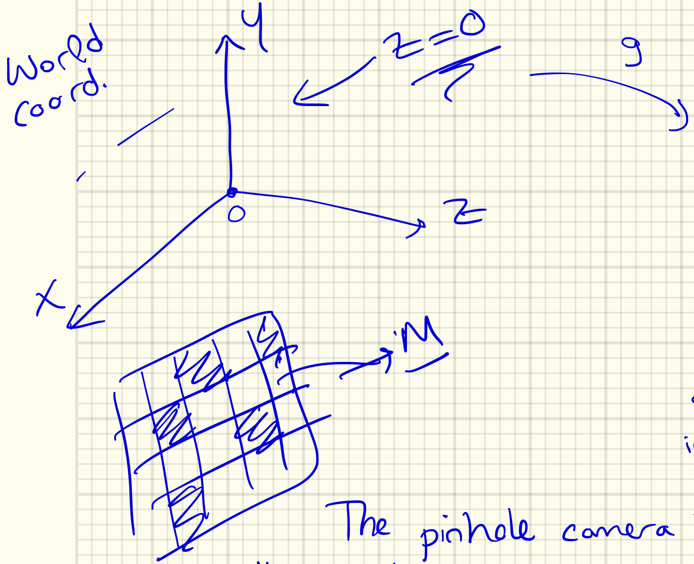
Radial Lens Distortion Model:

$$\breve{u} = u + (u - u_0)\left[k_1(x^2+y^2) + k_2(x^2+y^2)^2\right]$$

$$\breve{v} = v + (v - v_0)\left[k_1(x^2+y^2) + k_2(x^2+y^2)\right]$$

Lens Distortion Calibration $\Rightarrow$ Estimate k_1 & k_2.

Zhang's Camera Calibration: (Z. Zhang Technical Report)



Notation:

$$\underline{M} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \Rightarrow \tilde{M} = \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

3D model point

$$\underline{m} = \begin{pmatrix} u \\ v \end{pmatrix} \longrightarrow \tilde{m} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

2D image point

The pinhole camera is modeled by linking 3D point M & its projection m :

$$s\,\tilde{m} = \underline{\underline{A}}\;\underbrace{\begin{bmatrix} \underline{\underline{R}} & \underline{t} \end{bmatrix}}_{\text{extrinsic parameters}}$$

intrinsic camera matrix $\underline{\underline{A}} = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$

α, β: scale factors

γ: skewness of the image axes.

$\tilde{u}$.

(u_0, v_0) : coord. of the principal point.

$\rightarrow$ Homography btw the $\overset{\text{calibration object}}{\text{model}}$ plane & its image $\qquad \boxed{\lambda\, \underline{x}' = \underline{K}\, \boxed{\Pi_0\, \bar{g}}\, X_0}$

Assume model plane is on $\underline{Z=0}$ of the world coord. frame.

$$\underbrace{s\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}}_{\text{2D}} = \underline{\underline{A}}\, \overbrace{\begin{bmatrix} \underline{r_1} & \underline{r_2} & \underline{r_3} & \underline{t} \end{bmatrix}_{3\times 4}}^{= \Pi_0 \bar{g}} \underbrace{\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}}_{\text{3D}} \quad \leftarrow \text{assumption}$$

$$s\,\tilde{m} = \underbrace{\underline{\underline{A}} \begin{bmatrix} \underline{r_1} & \underline{r_2} & t \end{bmatrix}}_{\underline{\underline{H}}\ 3\times 3} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \longrightarrow$$

$\underline{\underline{H}}$ 3x3 homography defined upto a scalar

$$s\,\tilde{m} = \underline{\underline{H}}\,\tilde{M}$$

$\rightarrow$ Estimate $\underline{\underline{H}}$ first. Given n model points $\{M_i\}_{i=1}^{n} \longleftrightarrow \{m_i\}_{i=1}^{n}$

(Appendix A)

model points $\longleftrightarrow$ image points.

Sec 2.3

$$\begin{bmatrix} \underline{h_1} & \underline{h_2} & \underline{h_3} \end{bmatrix} = \underset{\sim}{\lambda}\, \underline{\underline{A}} \begin{bmatrix} \underline{r_1} & \underline{r_2} & \underline{t} \end{bmatrix}$$

$$\bigstar \quad \boxed{\begin{pmatrix} u \\ v \end{pmatrix} = \hat{m}_i = \frac{1}{\underline{h}_3^T \underline{M}_i} \begin{pmatrix} \underline{h}_1^T \underline{M}_i \\ \underline{h}_2^T \underline{M}_i \end{pmatrix}} \qquad \Leftarrow \quad \hat{m} = \underline{\underline{H}}\, \hat{M}$$

$$\underline{\underline{H}} = [\underline{h}_1 \quad \underline{h}_2 \quad \underline{h}_3]$$

h_i : ith row of $\underline{\underline{H}}$

$$\min_{\underline{\underline{H}}} \sum_i \| m_i - \hat{m}_i \|^2$$

m_i: measured image points

$\hat{m}_i$: projection of 3D model points M_i onto the image.

Nonlinear optimization problem: Uses Levenberg-Marquardt (LM) Algorithm (in minpack) (matlab "lsqnonlin").

→ Get an initial condition (guess) for $\underline{h}$:

Note $\hat{m} = \begin{pmatrix} u \\ v \end{pmatrix}$ Rewrite ($\bigstar$) →

$$\left.\begin{aligned} u\, \underline{h}_3^T \tilde{M}_i &= h_1^T \tilde{M}_i \\ v\, h_3^T \tilde{M}_i &= h_2^T \tilde{M}_i \end{aligned}\right\}$$

$$\underbrace{\begin{bmatrix} \tilde{M}^T & \underline{0}^T & -u\tilde{M}^T \\ \underline{0}^T & \tilde{M}^T & -v\tilde{M}^T \end{bmatrix}_{2n\times 9}}_{\underline{\underline{L}}_{2n\times 9}} \underbrace{\begin{bmatrix} \underline{h}_1 \\ \underline{h}_2 \\ \underline{h}_3 \end{bmatrix}_{9\times 1}}_{\underline{h}_{9\times 1}} = \underline{0}$$

$$\boxed{\underline{\underline{L}}\, \underline{h} = 0} \Longrightarrow$$ Solution to this homogeneous system of equations is the eigen vector of $\underline{\underline{L}}^T \underline{\underline{L}}$ corresp. to the smallest eigenvalue.

$\rightarrow \underline{h}$ ✓

Note

$$\underline{\underline{L}}\,\underline{h} = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -u[x\ y\ 1] \\ 0 & 0 & 0 & x & y & 1 & -v[x\ y\ 1] \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}$$

$$\underline{\underline{H}} = [\underline{h_1}\ \ \underline{h_2}\ \ \underline{h_3}] \checkmark$$

Use constraints $[\underline{h_1}\ \ \underline{h_2}\ \ \underline{h_3}] = \lambda\,\underline{\underline{A}}\,[r_1\ \ r_2\ \ r_3]$

$$\underline{r_1} = \underline{\underline{A}}^{-1}\underline{h_1}$$

$$\underline{r_2} = \underline{\underline{A}}^{-1}\underline{h_2}$$

(1) $\underline{r_1}^T \underline{r_2} = 0$

(2) $r_1^T r_1 = r_2^T r_2 = I$

(1) $\underline{h_1}^T \underline{\underline{A}}^{-T}\underline{\underline{A}}^{-1}\underline{h_2} = 0$

(2) $h_1^T \underbrace{A^{-T} A^{-1}}_{B} h_1 = h_2^T \underbrace{A^{-T} A^{-1}}_{\underline{\underline{B}}} h_2$

$$\underline{\underline{B}} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{23} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

$$\underline{\underline{A}} = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \underline{\underline{A}}^{-T} \underline{\underline{A}}^{-1} = \underline{\underline{B}}$$

$$\underline{\underline{B}} = \begin{bmatrix} \frac{1}{\alpha^2} & -\frac{\gamma}{\alpha^2 \beta} & \\ \vdots & & \\ & & \end{bmatrix}_{3\times 3}$$

symmetric

Eq. 5 $\Rightarrow$

$$\underline{b} = \begin{bmatrix} B_{11} \\ B_{12} \\ \vdots \\ B_{33} \end{bmatrix}_{6\times 1}.$$

① × ②

$$\Rightarrow \underbrace{\underline{h_i}^T \underline{\underline{B}} \, h_j = V_{ij}^T \underline{b}}$$

$$V_{ij} = \begin{bmatrix} h_{i1} h_{j1} \\ h_{i1} h_{j2} + h_{i2} h_{j1} \\ \vdots \end{bmatrix}_{6\times 1}$$

$$\left.\begin{aligned} &① \quad h_1^T B h_2 = 0 \\ &② \quad h_1^T B h_1 - h_2^T B h_2 = 0 \end{aligned}\right\} \Rightarrow \underset{2\times 6}{\underline{V}} \; \underset{6\times 1}{\underline{b}} = \underline{0} \Rightarrow \begin{bmatrix} V_{12}^T \\ (V_{11} - V_{22})^T \end{bmatrix} \underline{b} = 0$$

h_i

$\underline{\underline{B}}$

$\Rightarrow$ Once $\underline{b}$ is estimated; α, β, γ, u_0, v_0 are estimated ✓

$\Rightarrow$ Now, all camera intrinsic matrix $\underline{\underline{A}}$ are known ✓

$\Rightarrow$ Now, extrinsics can be calculated.

$\Rightarrow$ ★ $\underline{r_1} = \lambda \underline{\underline{A}}^{-1} \underline{h_1}$, $\underline{r_2} = \lambda \underline{\underline{A}}^{-1} \underline{h_2}$, $\underline{r_3} = \underline{r_1} \times \underline{r_2}$

$$\Rightarrow \lambda = \frac{1}{\|A^{-1} h_1\|} \qquad \lambda$$

$$\underline{t} = \lambda \underline{\underline{A}}^{-1} \underline{h_3}$$

At this point, we have initial estimates for all intrinsic & extrinsic parameters ✓

Given n images of the model plane & m points on the model plane

Set up the optimization:

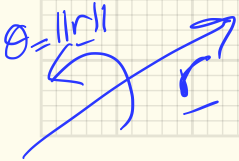
$$\min \sum_{\underset{n \text{ images}}{i=1}}^{n} \sum_{\underset{m \text{ points}}{j=1}}^{m} \| \underbrace{m_{ij}}_{\text{measurement } j \text{ on image plane } i} - \underbrace{\hat{m}(\underline{\underline{A}}, \underline{\underline{R}}_i, \underline{t}_i, \underline{M}_j)}_{\text{projection of point } M_j \text{ onto image } i} \|^2$$

Nonlinear optimization problem (eg. solve by LM Algo).

Bundle Adjustment $\leftarrow$

Note: $\underline{\underline{R}}$ is parameterized by $\underline{r}$: a 3D axis vector

its magnitude is the rotation angle



Using Rodrigues formula $\underline{\underline{R}} \Longleftrightarrow \underline{r}$