3D Vision

BLG634E

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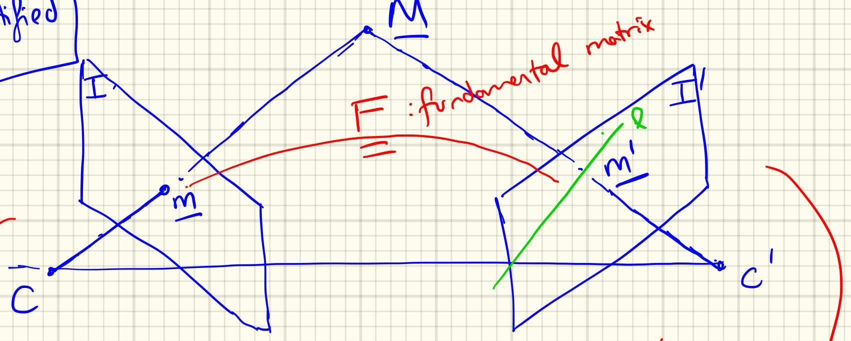
STEREO RECTIFICATION: [J. Mallon, P.F. Whelan] (Loop & Zhang)

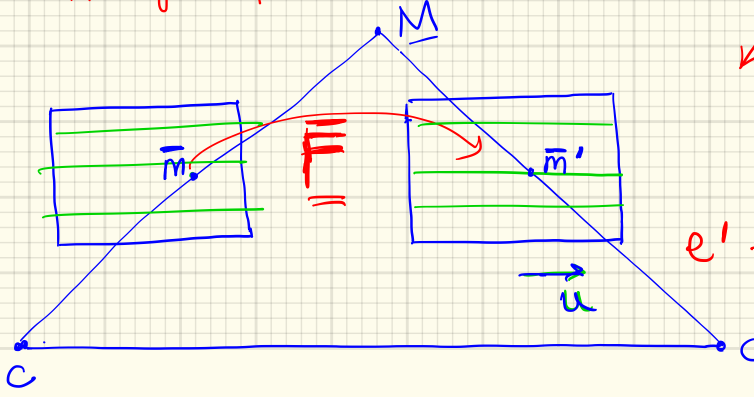
Transforming the 2 images in a stereo view by homographies to re-orient the epipolar lines so that they are parallel to the horizontal image axis.

Unrectified pair



H $\qquad$ $\underline{\underline{H'}}$

Rectified pair; epipoles are sent to ∞.



epipole is at ∞ ← e

$e' \rightarrow \infty$.

Given $\underline{\underline{F}}$, $\underline{m}$, $\underline{m}'$,

$$\underline{m}'^T \underline{\underline{F}}\, \underline{m} = 0$$

$$\boxed{\underline{\underline{F}}\, \underline{e} = \underline{0}}$$

$\underline{e}$ is in Null($\underline{\underline{F}}$)

$$\underline{\underline{F}}^T \underline{e}' = \underline{0}$$

Epipoles can be computed from SVD $\underline{\underline{F}} = \underline{\underline{U}}\,\underline{\underline{\Sigma}}\,\underline{\underline{V}}^T$

$$\underline{\underline{\Sigma}} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\underline{\underline{F}}^T \underline{\underline{F}} = V \Sigma \underbrace{U^T U}_{I} \Sigma V^T$$

$$\underline{\underline{F}}^T \underline{\underline{F}} = V \Sigma^2 \underbrace{V^T}_{V^{-1}}$$

Note: epipole e is the right singular vector corresp. to null singular value

→ u-coord : $\boxed{i = (1, 0, 0)^T}$

We want $\quad \underline{\underline{H}}\,\underline{e} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \underline{\underline{H}}'\,\underline{e}' = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

From the fundamental matrix property $\boxed{\underline{\underline{F}}\,\underline{e} = \underline{0}}$

$\underline{\underline{F}}$: fundamental matrix for the rectified pair

$$\underline{\underline{F}} \cdot \underline{i} = \underline{0} \quad \rightarrow \quad \text{Set} \quad \boxed{\underline{\underline{F}} \triangleq \hat{i} \triangleq \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}} = \underline{\underline{F}}$$

$$\hat{i}\, i = 0$$

$$\Rightarrow \underline{i} \times \underline{i} = 0$$

In the rectified stereo pair:

$$\underline{x}_2^T\, \underline{\underline{F}}\, \underline{x}_1 = 0 \quad ✓$$

$$\underbrace{\begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}} = \begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ y_1 \end{bmatrix} = 0 \Rightarrow \boxed{y_2 = y_1}$$

$\Longrightarrow$

In Rectified images:

1. Corresponding points are in the same rows.
2. All epipolar lines are parallel to the u-coordinate axis.

$$\underline{\underline{F}} \underbrace{\begin{pmatrix} u \\ v \\ 1 \end{pmatrix}} = \begin{pmatrix} 0 \\ -1 \\ v \end{pmatrix} \rightarrow l = (0, -1, v) \checkmark$$

$$\underline{\underline{F}} \cdot \underline{m} = \underline{l'}$$

eqn for this epipolar line

$l = (a, b, c)$

$ax + by + c = 0$

$-y + v = 0$

$\underline{y = v}$

Goal: Find $\underline{\underline{H}}$ & $\underline{\underline{H'}}$:

* The desired homographies give new image coord:

$$\underline{\bar{m}} = \underline{\underline{H}} \, \underline{m} \longrightarrow \text{1st image}$$

$$\underline{\bar{m}}' = \underline{\underline{H'}} \, \underline{m'} \longrightarrow \text{2nd image.}$$

Rectified Stereo:

$$\underline{\bar{m}}'^T \underline{\underline{\bar{F}}} \, \underline{\bar{m}} = 0 \quad \text{: epipolar constraint in rectified space.}$$

$$\underline{m}'^T \underbrace{\underline{\underline{H}}'^T \underline{\underline{\bar{F}}} \, \underline{\underline{H}}}_{\underline{\underline{F}}} \underline{m} = 0$$

$$(\star) \quad \boxed{\underline{\underline{F}} = \underline{\underline{H}}'^T \underline{\underline{\bar{F}}} \, \underline{\underline{H}}}$$

gives us a set of constraints relating H & H'.

Homographies satisfying $(\star)$ are not unique:

Choose an $\underline{\underline{H}}$ to transform the epipole to ∞:

$$\underline{\underline{H}} \, \underline{e} = \begin{pmatrix} e_u \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \longrightarrow \text{Set } \underline{\underline{H}} = \begin{array}{c} \\ h_{21} \\ h_{31} \end{array} \begin{bmatrix} 1 & 0 & 0 \\ -e_v/e_u & 1 & 0 \\ -1/e_u & 0 & 1 \end{bmatrix}$$

$$\underline{e} = \begin{pmatrix} e_u \\ e_v \\ 1 \end{pmatrix}$$

homogeneous coord.

Check $\boxed{\underline{\underline{H}} \begin{pmatrix} e_u \\ e_v \\ 1 \end{pmatrix} = \begin{pmatrix} e_u \\ 0 \\ 0 \end{pmatrix}}$

$$\rightarrow \underline{\underline{H}}' = \begin{bmatrix} 1 & 0 & 0 \\ h'_{21} & h'_{22} & h'_{23} \\ h'_{31} & h'_{32} & h'_{33} \end{bmatrix}$$

$$\underline{\underline{H}}'^T \underline{\underline{E}}\, \underline{H} = \underline{\underline{F}} \quad \star$$

$$\begin{pmatrix} 1 & 0 & 0 \\ h'_{21} & h'_{22} & h'_{23} \\ h'_{31} & h'_{32} & h'_{33} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ h_{21} & 1 & 0 \\ h_{31} & 0 & 1 \end{pmatrix} = F$$

$$= \alpha \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix}$$

Unknowns → stack into a vector:

$$\underline{P}_{7\times1} = (h'_{21}\ h'_{22}\ h'_{23}\ h'_{31}\ h'_{32}\ h'_{33}\ \alpha)_{7\times1}$$

$$\underbrace{\begin{bmatrix} -h_{31} & 0 & 0 & 1 & h_{21} & 0 & -f_{11} \\ 0 & 0 & 0 & 1 & 0 & 0 & -f_{12} \\ -1 & 0 & 0 & 0 & 0 & 0 & -f_{13} \\ 0 & -h_{31} & 0 & 0 & h_{21} & 0 & -f_{21} \\ \vdots & & & & & & \\ \vdots & & & & & & \end{bmatrix}_{9\times7}}_{\underline{\underline{B}}\ \checkmark} \underbrace{\begin{bmatrix} h'_{21} \\ h'_{22} \\ h'_{23} \\ h'_{31} \\ h'_{32} \\ h'_{33} \\ \alpha \end{bmatrix}_{7\times1}}_{\underline{P}\ \text{unknown}} = \underline{0} \qquad (\star\star)$$

Solve this in a least square sense.

using SVD of $\underline{\underline{B}}$ matrix

$$\equiv (\underline{B}^T \underline{\underline{B}}\ \text{EVD}) \rightarrow$$

$\underline{P}$: eigenvector corresp. to smallest evalue of $\underline{\underline{V}}$.

$\rightarrow$ $\underline{\underline{H}}$, $\underline{\underline{H}}'$ are solved, they rectify the stereo pair.

However, as 1^{st} rows of both homographies are arbitrary, (due to nature of the fundamental matrix)

$\rightarrow$ this introduces distortions in the rectified images.

$\sim$ $\hookrightarrow$ skewness.

Next step: Introduce $\underline{\underline{A}}$ s.t. $\underline{\underline{A}}\,\underline{\underline{H}}\,\underline{e} = \begin{pmatrix} e_u \\ 0 \\ 0 \end{pmatrix}$

epipole constraint is still satisfied.

$\therefore$ introducing $\underline{\underline{A}}$ does not change the constraint.

$$\underline{\underline{A}} = \begin{bmatrix} a_{11} & a_{12} & \boxed{a_{13}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow$ shift in x-dir.

estimate these new parameters

Let $\underline{\underline{K}} \triangleq \underline{\underline{A}}\,\underline{\underline{H}}$

$\underline{\underline{K}}' \triangleq \underline{\underline{A}}'\underline{\underline{H}}'$

Modify the homographies by these affine xforms,

$\rightarrow$ fundamental constraint:

$$H'^T A'^T \underline{\underline{F}} A H = \boxed{K'^T F K' = F}$$

we are free to specify the affine shearing xform that leaves the rectification unaffected.

$$\begin{cases} \begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{\underline{\underline{K}}} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} \\ \begin{pmatrix} x' \\ y' \end{pmatrix} \xrightarrow{\underline{\underline{K}}'} \begin{pmatrix} \bar{x}' \\ \bar{y}' \end{pmatrix} \end{cases}$$

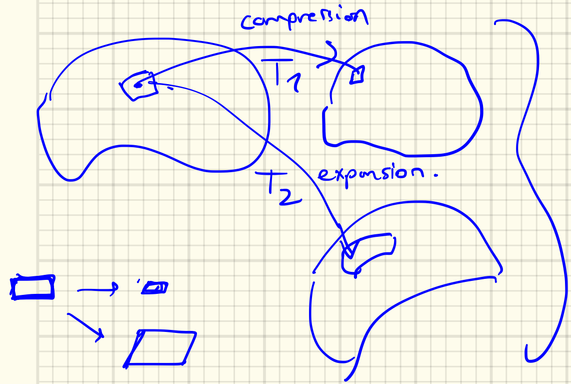
$\underline{\underline{K}}$, $\underline{\underline{K}}'$: final rectifying homographies

$$\left.\begin{aligned} K &= A\,H \\ K' &= A'\,H' \end{aligned}\right\} \rightarrow H, H' \text{ are known.}$$

Now, estimate parameters of $\underline{\underline{A}}$ matrices.

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ 1 \end{pmatrix} = \underline{\underline{A}}\; \underbrace{\underline{\underline{H}}}_{\text{known}} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \rightarrow$$

we will estimate a_{11}, a_{12}, as a_{13} does not introduce any distortion b/c a_{13} introduces an x-direction shift only.



$$\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{K} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} \rightarrow \underline{\underline{J}} = \begin{pmatrix} \dfrac{\partial \bar{x}}{\partial x} & \dfrac{\partial \bar{x}}{\partial y} \\ \dfrac{\partial \bar{y}}{\partial x} & \dfrac{\partial \bar{y}}{\partial y} \end{pmatrix}$$

coord. to coord. xform

Jacobian matrix for this coord. xform.

→ we can monitor singular values of $\underline{\underline{J}}$.

Digression

→ Let $\sigma_1(\underline{\underline{J}})$, $\sigma_2(\underline{\underline{J}})$ be non zero singular values of $\underline{\underline{J}}$, $\sigma_1 > \sigma_2$;

In general : $\sigma_1(\underline{\underline{J}}) > 1$ for a transform that expands → creates new pixels

$\sigma_1(\underline{\underline{J}}) < 1$ for a transform that compresses → destroys pixels

For an orthogonal transform ; that neither creates pixels nor destroys pixels, → singular values $= 1$.

Wielandt – Hoffman Theorem (See Golub, Van Loan) "Matrix Computation" book.

for singular values; states that if $\underline{\underline{A}}$ & $\underline{\underline{E}}$ are matrices in $\mathbb{R}^{m\times n}$, $m>n$

$$\sum_{k=1}^{n}\left(\sigma_k(\underline{\underline{A}}+\underline{\underline{E}})-\sigma_k(\underline{\underline{A}})\right)^2 \leq \|\underline{\underline{E}}\|_F^2$$: Frobenius norm of $\underline{\underline{E}}$

$\Rightarrow$ If $\underline{\underline{A}}$ is perturbed by $\underline{\underline{E}}$, the corresp. perturbation in any singular value of $\underline{\underline{A}}$ will be less than that of Frob. norm of $\underline{\underline{E}}$.

Recall: our goal to solve for a_{11}, a_{12} of $\underline{\underline{A}}$ matrix.

Search for a_{11} & a_{12} to maintain minimum distortion by Searching for singular values that are close to 1.

Setup an ⇒ Optimization problem:

$$\min f(a_{11}, a_{12}) = \sum_{i=1}^{n} \left(\sigma_1(\underline{\underline{J}}(K, \underline{p_i}) - 1\right)^2 + \left(\sigma_2(\underline{\underline{J}}(\underline{\underline{K}}, p_i) - 1\right)^2$$

search by evaluating singular values of the jacobian at various points $\underline{p_i}$ over the image.

→ This is minimized using Nelder-Mead (downhill) Simplex Method.

→ derivative-free optimization.

→ a_{11}, a_{12} are numerically solved.

(do the same for $'$ coord.)

⇒ K, K' are estimated.

DISPARITY Estimation (Stereo Matching):

Problem: Given a rectified stereo pair: Let I_l & I_R be the left & right intensity images, resp.

Given a pixel coord. in the left image,

→ Want to find the corresp. pixel coord. in the right image by minimizing a cost fn (let u be the disparity field),

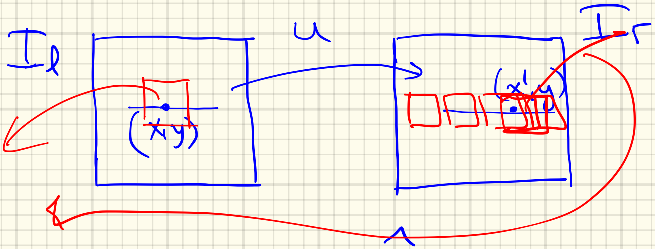
$$E(u) = \underbrace{E_{data}(u)}_{\text{measures how well the match is.}} + \lambda \cdot \underbrace{E_{\substack{regularizer\\smooth}}(u)}_{\text{e.g. enforces smoothness on the disparity field.}}$$

i) Data Cost: SSD: Sum of Squared Differences:

$$E_{Data}(u) = \sum_{(x,y)\in D} \Big[I_l(\underbrace{x,y}_{x,y}) - I_r(\underbrace{x-u(x,y)}_{(x',\ y)}, \underline{y}) \Big]^2$$

ii) Regularizer cost: (smoothness)

$$E_s(u) = \sum_{(x,y)\in D} |\nabla u(x,y)|^2$$



$\boxed{x' = x - u}$: shifting transform in x_dir.

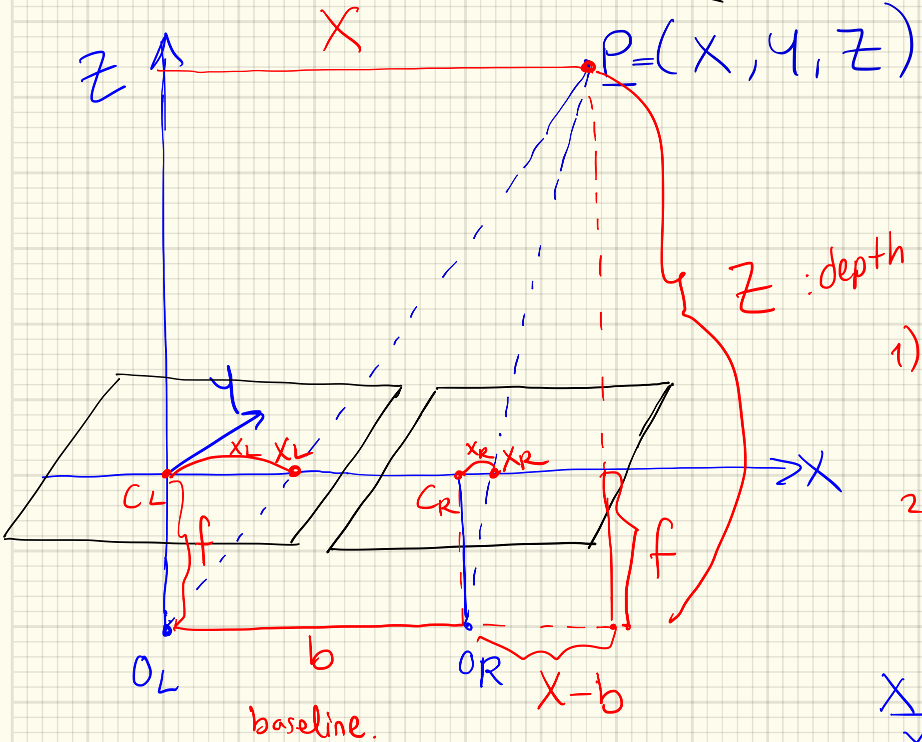
unknown disparity

Optimization: $\hat{u} = \arg\min_{u} E(u) = E_D(u) + \lambda E_s(u)$

→ Now, (after the optimization) we have estimated the disparity $\underset{\sim}{d}$. (or $\hat{u}$)

↳ how to estimate depth?

Depth Calculation for Rectified Cameras: from Disparity:



We know

$$\boxed{x_L - x_R \triangleq d}$$

disparity

Using similar triangles:

1) $$\frac{f}{x_R} = \frac{Z}{X-b}$$

2) $$\frac{Z}{f} = \frac{X}{x_L}$$

$$\frac{X-b}{x_R} = \frac{X}{x_L}$$

C_L, C_R : principal points

$$\frac{X-b}{X} = \frac{x_R}{x_L} \rightarrow 1 - \frac{b}{X} = \frac{x_R}{x_L} = \frac{b}{X} = 1 - \frac{x_R}{x_L}$$

$$\Rightarrow \quad X = \frac{b \cdot x_L}{\underbrace{x_L - x_R}_{d}}$$

$$\text{Depth } Z = f \cdot \frac{X}{x_L} = \frac{f}{x_L}\left(\frac{b \cdot x_L}{d}\right) = \frac{f\,b}{d}$$

$$\boxed{Z = \frac{f \cdot b}{d}}$$

baseline in meters.

disparity in pixels

focal length in pixels

Note: After depth Z is calculated,

$$X = x_L \frac{Z}{f}$$

$$Y = y_L \frac{Z}{f}$$

$$\rightarrow \underline{P} = (X, Y, Z)$$

3D Reconst.

Notes: 1) (x_L, y_L): not row, column directly, account for image centers (principal points)

$$x_L = \text{col} - c_L^x$$

$$y_L = \text{row} - c_L^y$$

2) (x_L, y_L) are in pixels

$\rightarrow X, Y, Z$ are in meters

Now, we completed the pipeline :

Simple pipeline:

1) Rectify images

2) For each pixel:

i. Find epipolar horizontal line

ii. Scan the line for "best match" → find d.

iii. Compute depth from disparity (d)

$$Z = \frac{b.f}{d}.$$

Reading Assignments:
- Stereo Rectification paper by Mallon, Whelan 2005
- Rectifying Homographies, Loop, Zhang 1999.